Homework 1

1. Prove Euler's formula for$\varphi(n)$using the formula of inclusions and exclusions.
2. Prove the equality

$$(x_1 + \cdots + x_k)^n = \sum_{m_1+\cdots+m_k=n} \frac{n!}{m_1! \dots m_k!} x_1^{m_1} \dots x_k^{m_k}$$

3. Calculate

$$\sum_{k=0}^{\lfloor n/3 \rfloor} \binom{n}{3k} \ , \quad \sum_{k=0}^{\lfloor n/3 \rfloor} \binom{n}{3k+1} \text{ and } \sum_{k=0}^{\lfloor n/3 \rfloor} \binom{n}{3k+2}$$

4. Calculate

$$\sum_{k=0}^{\lfloor n/4 \rfloor} \binom{n}{4k} \ , \quad \sum_{k=0}^{\lfloor n/4 \rfloor} \binom{n}{4k+1} \ , \quad \sum_{k=0}^{\lfloor n/4 \rfloor} \binom{n}{4k+2} \text{ and } \sum_{k=0}^{\lfloor n/4 \rfloor} \binom{n}{4k+3}$$

5*.Prove the equality

$$\sum_{k} \binom{n}{r+km} = \frac{2^n}{m} \sum_{k=1}^{m} \cos^n \frac{\pi k}{m} cos \frac{(n-2r)\pi k}{m}$$

6*.Find the number of the six-digit tickets with the sum of digits equal to 27.All positions in the ticket may be digits from [0,…, 9], i.e. the ticket may contain insignificant zeros.