## Final Exam: CGBL 2015

You have to solve 2 of 4 tasks from each section to pass exam.
Otherwise you should answer theoretical questions corresponding to the section you do not know by June, 17.

## Combinatorics (3)

№1

$$
\sum_{k}\binom{n}{3 k+2}=?
$$

№2 How many decimal sequences of a length N with no neighboring even numbers exist?
№3 Find a general solution of
$\left\{\begin{array}{l}x_{n}=x_{n-1}+y_{n-1}-1 \\ y_{n}=3 y_{n-1}+x_{n-1}+1\end{array}, x_{1}=y_{1}=2\right.$.
№4 Find a number of colorings in RGB palette for Tetrahedron Faces (with respect to all possible symmetries).

## Graphs (4)

№1 Prove that for every connected graph without loops and multiple edges there exists two vertices with the same degree value.


For a given graph:
№2 Identify vertices with local and global maximal descriptive characteristics and detect possible communities and graph cuts.
№3 Verify power-law and fit linear model with linear regression.
№4 Identify vertices with maximal values of centrality metrics and detect possible communities and graph cuts.

## Logic (3)

№1 Determine functional completeness of the system $\{0, x \oplus y \oplus z, x \equiv y\}$.
№2 Find finite system of axioms for [ $\{1, x \oplus y \oplus z, x \vee y\}$ ].
№3 Write a finite system of axioms for the closure of a system $\{\neg x, x \vee y\}$ based on a finite system of axioms constructed for full disjunctive normal form for generating system $\{\neg x, x \vee y, x \wedge y\}$.
№4 Find polynomial algorithm for 2-SAT problem - whether a given 2 -CNF is feasible (possess value 1 on some tuple).

