Homework 1, Digital Signal Processing class, HSE, Spring 2015. Due 30th of January 2015.
Instructor - Ossadtchi A., TA - Novikov N.

Problem 1

Stored in the memory of a digital signal processor is one cycle of the sinusoidal signal

$$x(n) = \sin\left(\frac{2\pi n}{N} + \theta\right)$$

where $\theta = 2\pi q/N$, where q and N are integers.

(a) Determine how this table of values can be used to obtain values of harmonically related sinusoids having the same phase.

(b) Determine how this table can be used to obtain sinusoids of the same frequency but different phase.

Problem 2

2.1 A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \frac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

(b) Sketch the signals that result if we:

(1) First fold $x(n)$ and then delay the resulting signal by four samples.

(2) First delay $x(n)$ by four samples and then fold the resulting signal.

(c) Sketch the signal $x(-n+4)$.

(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Problem 3

Operation called convolution of two sequences is defined as $z[n]=\sum_{k=-\infty}^{\infty} x[k]y[n-k]$

Compute sequence $z[n]$ for $n\in[-2,\ 5]$ if

a) $x[n]=-\delta[n+1]+\delta[n]+\delta[n-1]-2\delta[n-2]+3\delta[n-3]$, $y[n]=u[n-2]-u[n]$.

b) $x[n]=2\delta[n+1]-\delta[n]+\delta[n-1]-n\delta[n-2]-3\delta[n-3]$, $y[n]=\delta[n]$

c) Find $y[n]$ if $x[n]=\delta[n]+2\delta[n-1]-\delta[n-2]$ and $z[n]=-(u[n]-u[n-1])-2\delta[n-1]+\delta[n-2]$

Problem 4

One of the most basic random sequences is so called the sequence of independent identically distributed values, or in other words iid sequence. Each value in its realization is simply a random variable independently generated for each of the time slices and sampled from some probability density function (the same for each time-slice). Gaussian iid sequence is generated using Gausian pdf, say with mean $m=0$ and standard deviation $\sigma^2=1$.

Answer the following questions

a) Is this sequence a stationary random sequence? What is its mean?

b) The correlation sequence of the iid random sequence is $R[n]=\sigma^2\delta[n]$ Prove it!

Problem 5

In the lecture I considered the notion of ergodic processes. Clearly, an ergodic process must be stationary, but apparently not all stationary process is ergodic and therefore, not for every stationary process can we use averaging over time instead of averaging over the ensemble when computing the statistical moments. One example of a stationary bur not en ergodic process is the process $y[n]=w[n]+X$ where $w[n]$ is the Gaussian iid sequence and X is a Gaussian random variable. So, each m-th realization of the iid $w_m[n]$ is shifted by a realization of the random number X_m - the m-th realization of the random variable X.

a) Use matlab to generate 200 realizations of random process $y[n]$, say 256 time slices. You may want to visualize some of them using plot and imagesc commands.

b) Compute the mean values for each of the realization by averaging over time.

c) Compute the estimate of the mean sequence by averaging over the ensemble of 200 realizations.

d) Analytically prove that the process is not ergodic.