DSP class, Instructor Ossadtchi Alexei, TA – Novikov Nikita. Homework # 2, Due Friday, the 13th of February

1. Define odd part transform operating on s[n] and transforming it into its image S[n] = 0.5(s[n]- s[-n]). What are the basis vectors of the transform? Is the transform matrix invertible? Show that the transform matrix is rank deficient. Does Parseval's theorem hold? Why? Can you get back your signal from the coefficients of this transform.

2. Consider a periodic sequence such that $x[n+N]=x[n]$. Show that its *N*-point DFT is also periodic with period N

3. For N = 5 write out the explicit expressions for the 5 transform basis vectors, i.e. for the first basis vector (sequence) we will have $\boldsymbol{b}_0=\frac{1}{\sqrt{(5)}}[e^{j\frac{2\pi 0}{5}0}, e^{j\frac{2\pi 0}{5}1} ..., e^{j\frac{2\pi 0}{5}4}]=[1,1,...1]$
   1. How are $\boldsymbol{b}_1$ and $\boldsymbol{b}_3$ are related? Why?
   2. Explicitly compute all unique cross products and auto scalar products to show that the set of vectors (sequences) forms an othonormal basis
   3. Write out the transform matrix and its inverse
   4. Consider sequence $s[n]=[0,1,2,3,4]$.
      1. Write explicitly the expression for each of 5 coefficients of the the transform S[k], i.e. $S[0]=\frac{1}{\sqrt{(5)}}(1\times 0+1\times 1+1\times 2+1\times 3+1\times 4)=2\sqrt{(5)}$. Which property of the complex exponent would allow you to cheat here?
      2. Draw the real and imaginary parts of these coefficients.
      3. Which operation on the sequence s[n] would make S[0] = 0, why? Explain and test.
   5. Which family of real-valued sequences will have S[0], S[2],S[3] identically zero and nonzero S[1] and S[4].

4. Consider $\boldsymbol{b}_k=\frac{1}{\sqrt{(5)}}[1, e^{j\frac{2\pi k}{5}1}, e^{j\frac{2\pi k}{5}2}, e^{j\frac{2\pi k}{5}3} e^{j\frac{2\pi k}{5}4}]$ which is the k-th basis vector of the 5-point DFT. Why, by the way? In other words it is the vector made of the samples of the k-th basis sequence. In the ordinary DFT only the first five vectors indexed by k = 0,1,2,3,4 are used.
   1. Write out the explicit values for $\boldsymbol{b}_5$. Can you now guess and write out without calculations (but based on your results from the previous exercise (#2)) vectors $\boldsymbol{b}_{6,}..., \boldsymbol{b}_9$?
   2. Write out vector $\boldsymbol{v_1}=e^{-j\frac{2\pi k}{5}1}\boldsymbol{b}_k$, you may use complex exponent notation here, just make sure you simplify it and the power of the exponent is less than $2\pi k$
   3. Write out vector $\boldsymbol{v_2}=e^{-j\frac{2\pi k}{5}2}\boldsymbol{b}_k$. What do you observe?
   4. By now you should be able to cheat and write out $\boldsymbol{v_3}=e^{-j\frac{2\pi k}{5}3}\boldsymbol{b}_k$ without the explicit calculations. How about $\boldsymbol{v_4}=e^{-j\frac{2\pi k}{5}4}\boldsymbol{b}_k$.
   5. Describe your recipe for cheating. Can you apply it to write out $\boldsymbol{v_8}=e^{-j\frac{2\pi k}{5}8}\boldsymbol{b}_k$

5. Use Matlab and write a simple function that calculates the coefficients of the N-point DFT transform for an arbitrary sequence with N elements. Don't call it DFT, use some other personalized name.