DSP class, Instructor Ossadtchi Alexei, TA – Novikov Nikita. Homework # 2, Due Friday, the 6th of March (at the seminar)

1. There is a slang expression in the field stating that "Linear systems can not create new frequencies in the output data". What do you think it means? Explain and illustrate this statement with equations. To solve this problem you will need to recall the notion of system's transfer function and what happens to the Fourier coefficients when a signal is passed thought an LTI system.

2. At the lecture we have learned how to replace a pair of cascaded systems with pulse responses sequences $h_1[n]$ and $h_2[n]$ by a single system. Consider here linear (not circular) convolution. Does the order in which the two systems arranged matter? In other words, will the output for the same signal differ if you change the order of the two systems in the cascade?
   1. Prove it analytically, using the expression for convolution
   2. Write a simple matlab code and verify your conclusion.
   3. Now, switch to the circular convolution and repeat the above

3. Represent circular convolution of $h[n]$ and $x[n]$ as a product of matrix **H** created with properly arranged elements of $h[n]$. Do so for the particular case of sequence of length N = 5.

4. Based on the previous exercise and using your linear algebra intuition
   1. Can you guess how we could define the system inverse with respect to the system with pulse response $h[n]$? Write down your thoughts here.
   2. For a perfect delay system the pulse response is $h[n]=\delta[n-d]$. Assume d = 1, and write matrix $\boldsymbol{H}$ for sequences of length N = 5. Don't forget about the circularity of the convolution operation. Find the transformation matrix $\boldsymbol{H}_{inv}$ for the system inverse to the perfect delay system. Is this inverse system causal?
   3. Does every system have its inverse. How can you check this?