HSE DSP (Signals and systems class), Homework 5, due the 27th of March 2015.

Task 1

Write a simple Matlab script to prove that the autocorrelation sequence of white noise random process is delta pulse scaled by the variance of the random process. Use randn(1,1000) function to generate a realization of the random process (pseudo random process to be precise). Compute 100 lags of the autocorrelation sequence. Make necessary plots.

Task 2

Suppose, that x is a wide sense stationary process with autocorrelation sequence $R[n] = Ae^{-bn}$.

1. Draw this autocorrelation sequence for different values of *b*.
2. What is the variance of this random process?
3. What is the limit of R[n] as n goes to infinity? Does this process have zero mean value?
4. What is the correlation of the two random variables x[4] and x[6]?
5. The second moment of a random variable *z* is defined as the expected value of the square of this variable , i.e. $E\{z^2\}$. Determine the second moment of random variable $z = x[6] - x[2]$
6. What is the total power of this process? To answer this use DTFT tables and first calculate the power spectral density of this process

Task 3

Narrow-band random process is widely used in many applications, from telecommunication to analysis of EEG data. Model power spectral density $S_X(\omega)$ of this process is a pair of symmetric with respect to $\omega = 0$ narrow rectangular blocks over the central frequency ω_0 and $-\omega_0$. IN other words

$$S_X(\omega) = \begin{cases} \frac{1}{2}, |\omega - \omega_0| < \Delta\omega/2 \\ 0, otherwise \end{cases}$$

The "narrowbandness" index is the ratio of $\Delta\omega/\omega_0$ and the smaller this value is the more the process resembles a simple harmonic oscillation , the more narrowband it is. Draw $S_X(\omega)$.

1. What is the autocorrelation function of this process? Use Wiener-Khintchine theorem to derive and draw it.
2. The process can be represented as a harmonic oscillation $z[n]$ with frequency ω_0 , called carrier, multiplied by a slowly varying process $e[n]$, called envelope ("ogibayuschaya"). Assume that the envelope and the carrier are statistically independent. In this case the expression for the autocorrelation function of this process can be written as $R_X[k] = E\{z[n]e[n]z[n+k]e[n+k]\} = E\{z[n]z[n+k]\}E\{e[n]e[n+k]\} = R_e[n]R_z[n]$ where $R_e[k]$ and $R_z[k]$ are the autocorrelation sequences of the envelope and the carrier correspondingly. Note that statistical independence of the carrier and the envelope allowed us to split the expectation operation of the product into the product of two expectations. What is the power spectral density function of the envelope? Of the carrier? To answer this question
    a. Represent $S_X(\omega)$ as a convolution of two delta functions and $rect_{\Delta\omega/2}(\omega)$ function.
    b. Use the convolution theorem to argue and write the expression for the power spectral density of the envelope and the carrier.
    c. What is the autocorrelation sequence of $e[n]$
3. Suggest a way to generate a realization of this random process
4. Write a simple Matlab script and generate a 1000 samples long realization of the narrowband random process *X* for $\omega_0 = 0.5\pi$ and $\Delta\omega/2 = 0.05$
5. Write a simple script to calculate the estimate of the autocorrelation sequence $R[n]$.
6. Use Wiener-Khintchine theorem and obtain the power spectral density of this random process.
7. Given a realization of this random process can you suggest a way to extract the envelope? This operation is routinely performed by an AM radio. Can you do it?