VIII Open Autumn university Students' Mathematical Competition
OSAM Comp 2025 by FCS HSE

September 14th 2025, 11:00 am – 2:00 pm UTM +3

Category I (I year bachelors)

1. As is well known, the number 25 when squared ends with the same two digits: **625**. Find all two-digit numbers in base six that have the same property: that is, when the square of this number is written in base six, the last two digits of the representation coincide with the original number (also in base 6).

2. Find the last digit of the number $\left[\dfrac{10^{200}}{10^{10}+3}\right]$.

3. For every real $x \geqslant \frac{1}{3}$ we define the function $f(x)$ as
   $f(x) = x + \cfrac{1}{3x + \cfrac{1}{3x + \cfrac{1}{3x + \ddots}}}$. Compute $f(0.5)f'(0.5)$.

4. All cells of an $n \times n$ rectangular table are filled with zeros and ones. It turned out that the cells of the main diagonal (that is, the cells M_{ii} at the intersection of the i-th row and the i-th column for each i from 1 to n) contain only zeros, and in each pair of cells symmetric with respect to the main diagonal (that is, the pair of cells M_{ij} and M_{ji} for distinct indices i and j), exactly one contains a one. The sum of the numbers in the i-th row is denoted by r_i, and the sum of the numbers in the j-th column is denoted by c_j. Prove that

$$\sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} c_i^2.$$

5. Let the function $\rho(\alpha, \beta)$ be defined as

$$\rho(\alpha, \beta) = \max\{\sin\alpha\sin\beta, 1-\sin\alpha-\sin\beta+\sin\alpha\sin\beta, \sin\alpha+\sin\beta-2\sin\alpha\sin\beta\}$$

   for arbitrary real α and β. Find the minimum of this expression.