VIII Open Autumn university Students' Mathematical Competition
OSAM Comp 2025 by FCS HSE

September 14th 2025, 11:00 am – 2:00 pm UTM +3

Category II+ (2nd year bachelors and further)

1. Polynomials F and G with real coefficients are such that $F(F(x)) > F(G(x)) > G(G(x))$ for all real x. Prove that $F(x) > G(x)$ for all real x.

2. For a regular n-gon M_n with side length 1, we call the *outer angularity* E_n the area inside its circumscribed circle but outside M_n. We call the *inner angularity* I_n the area inside M_n but outside its inscribed circle. Find the limit of E_n/I_n as n tends to infinity.

3. Does there exist a 3×3 matrix X (over $\mathbb{C}$) such that the following equality holds:

$$X^2 - X \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}?$$

4. In the plane, two parabolas are given: the parabola Γ_0 is defined by the equation $y = -x^2$ and is fixed. Another parabola Γ_1 is initially defined by the equation $y = x^2$, thus touching the parabola Γ_0. Γ_1 can move without slipping along the surface of the fixed parabola Γ_0 (while rotating). Find the locus of the foci of the parabola Γ_1 under such motions.

5. Let S be a set of points in $\mathbb{R}^{2025^{2025}}$ containing $D = 2025^{2025^{2025}}$ points, and let $f : S \to \mathbb{R}$ be a mapping. Denote by Mf the median of the set of values f on the elements of S, and by $\overline{f}$ the arithmetic mean of these values. It is known that for any positive $t > 0$, the number of points satisfying the inequality $|f(x) - Mf| > t$ does not exceed $e^{-8t^2}D$. Show that for any positive $t > 0$, the number of points satisfying the inequality $|f(x) - \overline{f}| > t$ does not exceed $3e^{-t^2}D$.