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Abstracts of talks

Abstracts are listed in alphabetical order of the speakers. The list of speakers can be found in the last page of this document (page 32).

Substitution complexes and nonrealizability example

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Abstract: We discuss the question of realisability of iterated higher Whitehead products with a given form of nested bracket by simplicial complexes, using the notion of the moment-angle complex $\mathcal{Z}_{\mathcal{K}}$. For an iterated higher Whitehead product w we describe a simplicial complex $\partial\Delta_w$ that realises w (see $\partial\Delta_{[[\mu_1, \mu_2, \mu_3], \mu_4, \mu_5]}$ in Fig. 1). Furthermore, for a particular form of brackets inside w , we prove that $\partial\Delta_w$ is the smallest complex that realises w . For certain w , we show that $\mathcal{Z}_{\mathcal{K}}$ is homotopy equivalent to a wedge of spheres and each sphere is a lift of an iterated higher Whitehead product. For example, for $\mathcal{K}$ in Fig. 1, the corresponding moment-angle complex $\mathcal{Z}_{\mathcal{K}}$ is homotopy equivalent to $(S^5)^{\vee 4} \vee (S^6)^{\vee 3} \vee S^7 \vee S^8$ (see [1], [2]). It is not true that if $\mathcal{Z}_{\mathcal{K}}$ splits into a wedge of spheres, then each sphere is a lift of a Whitehead product. We present an example of this situation (see [1]).

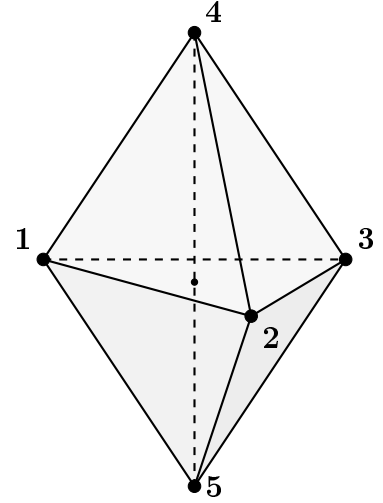


Figure 1: $\partial\Delta_{[[\mu_1, \mu_2, \mu_3], \mu_4, \mu_5]}$

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Commutative algebraic monoids and toric geometry

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Abstract: An (affine) algebraic monoid is an irreducible (affine) algebraic variety S with an associative multiplication

$$\mu: S \times S \rightarrow S, \quad (a, b) \mapsto ab,$$

which is a morphism of algebraic varieties, and a unit element $e \in S$. Examples of affine algebraic monoids are affine algebraic groups and multiplicative monoids of finite dimensional associative algebras. The group of invertible elements $G(S)$ of an algebraic monoid S is open in S . Moreover, $G(S)$ is an algebraic group. By a result of Rittatore, every algebraic monoid S , whose group of invertible elements $G(S)$ is an affine algebraic group, is an affine monoid. An affine algebraic monoid S is called reductive if $G(S)$ is a reductive algebraic group.

By a group embedding we mean an irreducible affine variety X with an open embedding $G \hookrightarrow X$ of an affine algebraic group G such that both actions by left and right multiplications of G on itself can be extended to G -actions on X . In other words, the variety X is a $(G \times G)$ -equivariant open embedding of the homogeneous space $(G \times G)/\Delta(G)$, where $\Delta(G)$ is the diagonal in $G \times G$.

Any affine monoid S defines a group embedding $G(S) \hookrightarrow S$. The converse statement claims that for every group embedding $G \hookrightarrow S$ there exists a unique structure of an affine algebraic monoid on S such that the group G coincides with the group of invertible elements $G(S)$. This is proved in [5] under the assumption that G is reductive and in [4] for arbitrary G .

We study commutative affine algebraic monoids. The ground field $\mathbb{K}$ is an algebraically closed field of characteristic zero. The theory of commutative reductive algebraic monoids, i.e. algebraic monoids with an algebraic torus as the group of invertible elements, is nothing but the theory of affine toric varieties. We concentrate on non-reductive commutative monoids. Here a crucial role plays the notion of a Demazure root of an affine toric variety.

Let us define the rank of a commutative monoid S as the dimension of the maximal torus of the group $G(S)$. There is a classification of commutative monoids of rank 0, $n - 1$ and n . In particular, this gives a classification of commutative monoid structures on $\mathbb{A}^n$ for $n \leq 2$. More generally, a classification of commutative monoid structures on normal affine surfaces is given in [2]. Every surface admitting a structure of a commutative monoid turns out to be toric.

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In our main result in [1] we classify commutative monoid structures on $\mathbb{A}^3$. For $b, c \in \mathbb{Z}_{\geq 0}$, $b \leq c$, denote by $Q_{b,c}$ the polynomial

$$Q_{b,c}(x_1, y_1, x_2, y_2) = \sum_{k=1}^d \binom{d+1}{k} x_1^{e+b(k-1)} y_1^{e+b(d-k)} x_2^{d-k+1} y_2^k,$$

where $c = bd + e$, $d, e \in \mathbb{Z}$, $0 \leq e < b$. Note that $Q_{b,c}(x_1, y_1, x_2, y_2) = Q_{b,c}(y_1, x_1, y_2, x_2)$ and

$$Q_{b,c}(x_1, y_1, x_2, y_2) = \frac{(x_1^b y_2 + y_1^b x_2)^{d+1} - (x_1^b y_2)^{d+1} - (y_1^b x_2)^{d+1}}{x_1^{b-e} y_1^{b-e}}.$$

Theorem. Every commutative monoid on $\mathbb{A}^3$ is isomorphic to one of the following monoids:

rk	Notation	$(x_1, x_2, x_3) * (y_1, y_2, y_3)$
0	$3A$	$(x_1 + y_1, x_2 + y_2, x_3 + y_3)$
1	$M \begin{smallmatrix} + \\ b \end{smallmatrix} A \begin{smallmatrix} + \\ c \end{smallmatrix} A$	$(x_1 y_1, x_1^b y_2 + y_1^b x_2, x_1^c y_3 + y_1^c x_3),$
1	$M \begin{smallmatrix} + \\ b \end{smallmatrix} A \begin{smallmatrix} + \\ b, c \end{smallmatrix} A$	$(x_1 y_1, x_1^b y_2 + y_1^b x_2, x_1^c y_3 + y_1^c x_3 + Q_{b,c}(x_1, y_1, x_2, y_2)),$
2	$M \begin{smallmatrix} + \\ b, c \end{smallmatrix} M \begin{smallmatrix} + \\ b, c \end{smallmatrix} A$	$(x_1 y_1, x_2 y_2, x_1^b x_2^c y_3 + y_1^b y_2^c x_3),$
3	$3M$	$(x_1 y_1, x_2 y_2, x_3 y_3)$

Moreover, every two monoids of different types or of the same type with different values of parameters from this list are non-isomorphic.

The proof is based on a classification of pairs of commuting homogeneous locally nilpotent derivations of degree zero on a positively graded polynomial algebra $\mathbb{K}[x_1, x_2, x_3]$.

Key words and phrases: algebraic group, algebraic monoid, grading, toric variety, Demazure root.

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B-root subgroups on affine spherical varieties

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Abstract: In the study of automorphism groups of toric varieties, a key role is played by one-parameter additive groups normalized by the acting torus. Such subgroups are called root subgroups and each of them is uniquely determined by its weight, called the Demazure root of the corresponding toric variety. Moreover, the set of all Demazure roots admits an explicit combinatorial description in terms of the fan defining the toric variety, and this description is especially simple in the case when the variety is affine.

In the setting of arbitrary connected reductive groups acting on algebraic varieties, a natural generalization of toric varieties is given by spherical varieties. A spherical variety is an algebraic variety X equipped with an action of a connected reductive group G in such a way that a Borel subgroup B of G has an open orbit in X . It turns out that a proper generalization of root subgroups for spherical varieties is given by one-parameter additive groups normalized by B , which we call B -root subgroups. In the talk we shall discuss B -root subgroups on affine spherical varieties, including basic properties, applications, and open problems.

The talk is based on a work in progress joint with Ivan Arzhantsev.

On the cohomology of polyhedral products — multiplicative results

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Abstract: The problem of computing the cohomology of a polyhedral product has an extensive history. Some of the earliest results in this direction, for the case of moment-angle complexes, appeared in the work of M. Franz, I. Baskakov, V. Buchstaber and T. Panov and also S. López de Medrano. There is also a literature of results for other specific and general CW pairs which I shall review briefly.

This talk is a report on the recent completion of a project initiated ten years ago. Our goal is an explicit description of the additive and multiplicative structure of the cohomology of a polyhedral product. The result, for field coefficients and general CW pairs, has a transparency sufficient to allow for direct computation. The focus in this lecture will be on the multiplicative results. This is joint work with M. Bendersky, F.R. Cohen and S. Gitler.

Coloring by 15 colors and small covers in dimension 4

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Abstract: In general, for an n -dimensional simple polytope P , when $n > 3$ it is not known whether it admits the structure of a small cover or a quasitoric manifolds. However, the following estimation is known [1] for the chromatic number of a simple polytope P admitting a small cover (quasitoric manifold)

$$n \leq \chi(P) \leq 2^n - 1,$$

in sense that $\chi(P)$ is the minimal number of colors necessary to paint the facets of P regularly. In dimension 2, $\chi(P)$ is equal 2 or 3, while in dimension 3, $\chi(P)$ is equal 3 or 4. However, in higher dimensions we do not know if the upper bound in the upper inequality is sharp or not.

In the talk, we construct a simple 4-polytope admitting a small cover and whose chromatic number is $15 = 2^4 - 1$, the predicted upper bound. The same polytope also admits a quasitoric manifold. Now a natural question arises: *For $n \geq 4$ does there always exist a simple n -polytope P admitting a small cover such that $\chi(P) = 2^n - 1$?*

Key words and phrases: small cover, quasitoric manifold, simple polytope, chromatic number.

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Singularities of T^n -action on the Grassmannians $G_{n,2}$ and their universal resolution

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Abstract: The talk is devoted to the classical problem about the structure of the standard action of the compact torus T^n on the complex Grassmann manifold $G_{n,2}$, which is closely related to series of problems in algebraic topology, algebraic geometry and mathematical physics.

We use the known moment map $\mu : G_{n,2} \rightarrow \Delta_{n,2}$ which gives the projection $\hat{\mu} : X_n \rightarrow \Delta_{n,2}$, where $\Delta_{n,2}$ is the hypersimplex and $X_n = G_{n,2}/T^n$.

In the base of the proposed approach is the notion of the singular point of the orbit space $G_{n,2}/T^n$ for which the set $Y_n = X_n \setminus \text{sing} X_n$ is an open algebraic manifold, where $\text{sing} X_n$ is the set of singular points. We discuss the relation between the set $\text{sing} X_n$ and the set of critical points of the moment map as well as with the set of the points from $G_{n,2}$ which have the non-trivial stationary subgroup.

Our main result is the construction of the smooth, compact manifold $\mathcal{F}_n$ called the universal space of parameters by which we define continuous, surjective map $G_n : \Delta_{n,2} \times \mathcal{F}_n \rightarrow G_{n,2}/T^n$ such that the map $G_n : G_n^{-1}(Y_n) \rightarrow Y_n$ is a diffeomorphism and $\hat{\mu}(G_n(x, f)) = x$.

By the same construction it is given the description of the sets $G_n^{-1}(z)$ for all $z \in \text{sing} X_n$. Note that this description essentially uses the fact that our resolution of singularities of the space $X_n = G_{n,2}/T^n$ is functorial for the canonical equivariant embeddings of the manifolds $G_{n-1,2}$ and $G_{n,1}$ into the manifold $G_{n,2}$.

The talk is based on joint results with Svjetlana Terzić.

Key words and phrases: torus action, Grassmann manifolds, singular points.

Torsion free small covers

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Abstract: It is well known that if a small cover is a pullback of the linear model in the sense of [1], then it is torsion free. In this talk we will show the the converse is also true: if a small cover is toresion free then it must be a pullback of the linear model. This is a joint work with Suyoung Choi and Hanchul Park.

Details can be found in [2].

Key words and phrases: small cover, pullback of the linear model

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The cohomology of Gelfand–Zeitlin fibers

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Abstract: Gelfand–Zeitlin systems are a well-known family of examples in symplectic geometry, singular Lagrangian torus fibrations whose total space is a coadjoint orbit of a unitary group and whose base space is a certain convex polytope. They are easily defined in terms of matrices but do not fit into the familiar framework of integrable systems with non-degenerate singularities, and hence are much studied.

Despite the prominence of Gelfand–Zeitlin systems, not much has been known about the topology of their fibers. In this talk, we compute their integral cohomology rings.

This represents joint work with Jeremy Lane.

Topology of orbit spaces of complexity one torus actions in non-general position

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Abstract: In this talk, I am going to discuss the results of the paper [1] which is devoted to orbit spaces complexity one torus actions on smooth manifolds. I will present the main result for actions not in general position. It turns out that for such action, the homology groups of the orbit space can be arbitrary in dimensions 3 and higher and, given that the action is Hamiltonian, homology groups of lower dimensions are trivial. I will also propose concrete examples of manifolds with such actions – namely, manifolds of isospectral Hermitian matrices and regular semisimple Hessenberg varieties.

Preliminary knowledge will be given during the talk. For further details, see the reference list.

Key words and phrases: torus actions, complexity one, Hessenberg varieties.

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Gromov width of symplectic toric manifolds associated with graph associahedra

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Abstract: A graph associahedron is a nestohedron corresponding to a simple graph. It can be regarded as a Delzant polytope, and, hence, there is the associated symplectic toric manifold. The Gromov width of a symplectic manifold is an invariant that measures the maximal size of a ball which can be symplectically embedded. We give an explicit formula for the Gromov width of symplectic toric manifolds obtained from connected simple graphs. This talk is based on a joint work with Taekgyu Hwang.

Geometrization of 3-manifolds realizable as small covers

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Abstract: Geometrization conjecture of W. Thurston (finally proved by G. Perelman) says that *any oriented closed 3-manifold which is prime with respect to the connected sum operation, can be cut along tori so that the interior of each of the resulting manifolds has a geometric structure with finite volume of one of the 8 types*. M. W. Davis and T. Januszkiewicz in Example 1.21 of [1] considered a special class of 3-manifolds, which are small covers over 3-polytopes. They sketched a proof of Thurston's conjecture for manifolds from this class

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using Andreiev's theorem. It turned out that some statements in their exposition need to be corrected. We show that for this one needs to use the notion of almost Pogorelov polytopes, which is not mentioned there. Thus, in fact, [1, Example 1.21] led to a problem. We give a solution of this problem (see [2]).

We use details from the theory of 3-manifolds following [3]. A *connected sum* of closed oriented 3-manifolds N_1 and N_2 is defined as $N_1 \# N_2 := (N_1 \setminus \text{int } B_i) \cup_f (N_2 \setminus \text{int } B_2)$, where for embedded 3-balls $B_i \subset N_i$ the boundary ∂B_i is endowed with the orientation induced from $N_i \setminus \text{int } B_i$, and $f: \partial B_1 \rightarrow \partial B_2$ is an orientation-reversing homeomorphism. It can be shown that the homeomorphism type of $N_1 \# N_2$ does not depend on the choice of B_i and f . An orientable 3-manifold N is called *prime* if it cannot be decomposed as a non-trivial connected sum of two manifolds, that is, if $N = N_1 \# N_2$, then $N_1 = S^3$ or $N_2 = S^3$. A 3-manifold N is called *irreducible* if every embedded 2-sphere in N bounds a 3-ball in N . An orientable, irreducible 3-manifold is prime. Conversely, if N is an orientable closed prime 3-manifold, then either N is irreducible or $N = S^1 \times S^2$. The sphere S^3 is irreducible.

Theorem (Kneser-Haken-Milnor) Let N be a compact oriented closed 3-manifold. There exist oriented prime 3-manifolds $N_1, \dots, N_m$ such that $N = N_1 \# \dots \# N_m$. Moreover, if $N = N_1 \# \dots \# N_m$, and $N = N'_1 \# \dots \# N'_n$, with oriented prime 3-manifolds N_i and N'_i , then $m = n$ and (possibly after reordering) there exist orientation-preserving homeomorphisms $N_i \rightarrow N'_i$.

By a *polytope* we mean a combinatorial convex 3-polytope. Boundaries of polytopes are in bijection with combinatorial types of simple spherical graphs with all facets bounded by simple cycles and any nonempty intersection of two facets being either a vertex or an edge. A *k-belt* of a 3-polytope is a cyclic sequence of k -facets such that facets are adjacent if and only if they follow each other, and no three facets have a common vertex. A simple 3-polytope P different from the simplex Δ^3 is *flag*, if it has no 3-belts. For a flag 3-polytope P its real moment-angle manifold $\mathbb{R}Z_P$ and any orientable small cover $M(P, \Lambda)$ is *aspherical* (that is, $\pi_i(X) = 0$ for $i \geq 2$). Any aspherical compact oriented closed 3-manifold is irreducible. A *connected sum* of simple 3-polytopes P and Q along *vertices* is a polytope such that its boundary arises from boundaries of P and Q if we cut the corresponding vertices of P and Q , glue the complements to the arising triangles in ∂P and ∂Q along their boundaries and delete the 3-cycle from the graph. The following result is known.

Theorem 1. 1) Any simple 3-polytope P can be uniquely decomposed into a connected sum along vertices of 3-polytopes, such that each polytope is either Δ^3 or a flag 3-polytope. 2) For $\mathbb{R}Z_P$ this corresponds to the prime decomposition into a connected sum of copies of S^3 , $S^1 \times S^2$, and manifolds $\mathbb{R}Z_Q$ of flag 3-polytopes Q . 3) For any orientable small cover $M(P, \Lambda)$ this corresponds to the prime decomposition into a connected sum of copies of $\mathbb{R}P^3$ and small covers over flag 3-polytopes.

A *Seifert fibered manifold* is a compact 3-manifold N together with a decomposition of N into disjoint simple closed curves (called Seifert fibers) such that each Seifert fiber has a tubular neighborhood that forms a standard fibered torus. The *standard fibered torus* corresponding to a pair of coprime integers $(a; b)$ with $a > 0$ is obtained from the cylinder $D^2 \times I$, if we identify its bases by the rotation by the angle $2\pi b/a$. It has a natural fibering by circles.

A 2-dimensional closed submanifold is called *incompressible*, if its inclusion induces injection on π_1 . A 3-manifold N is *atoroidal* if any map $T \rightarrow N$ from a torus to N which induces a monomorphism $\pi_1(T) \rightarrow \pi_1(N)$ can be homotoped into the boundary of N . Orientable hyperbolic manifold of finite volume is atoroidal. The following theorem was proved independently by Jaco–Shalen and Johannson.

Theorem. (JSJ-decomposition) *Let N be a closed compact orientable irreducible 3-manifold. There exists a (possibly empty) collection of disjointly embedded incompressible tori $T_1, \dots, T_m$ such that each component of N cut along $T_1, \dots, T_m$ is atoroidal or Seifert fibered. Any such collection of tori with a minimal number of components is unique up to isotopy.*

An *almost Pogorelov polytope* is a flag 3-polytope with any 4-belt surrounding a facet. If P is not a k -prism, then all its quadrangles are disjoint and Andreev’s theorem implies that if we shrink them to points, the arising polytope $\hat{P}$ is realizable as a right-angled polytope of finite volume in the Lobachevsky (hyperbolic) space $\mathbb{H}^3$. The quadrangles of P correspond to ideal vertices of $\hat{P}$. Details see in [4]. A *connected sum* of flag 3-polytopes P and Q along *quadrangles* is a polytope such that its boundary arises from boundaries of P and Q if we glue the complements to quadrangles in ∂P and ∂Q along the boundaries and delete the arising 4-cycle. Our main result is the following.

Theorem 2. 1) *Any simple flag 3-polytope $P \neq I^3$ can be uniquely decomposed into a connected sum along quadrangles of k -prisms $k \geq 5$ and almost Pogorelov polytopes such that for any two prisms glued along quadrangles their bases are not glued along edges.*
 2) *For $\mathbb{RZ}_P$ this corresponds to the minimal JSJ-decomposition: each 4-belt gives a collection of incompressible tori, each k -prism corresponds to a Seifert fibered manifold*

$$(\text{oriented surface with holes}) \times S^1$$

and carries a metric of $\mathbb{H}^2 \times \mathbb{R}$ with finite volume, and each almost Pogorelov polytope corresponds to a hyperbolic manifold of finite volume.

3) *For any orientable small cover over P this corresponds to the minimal JSJ-decomposition: each 4-belt gives 1 or 2 incompressible tori, each k -prism corresponds to a Seifert fibered manifold and carries a metric of $\mathbb{H}^2 \times \mathbb{R}$ with finite volume, and each almost Pogorelov polytope corresponds to a hyperbolic manifold of finite volume.*

4) *For non-orientable small covers this gives a collection of incompressible tori and Klein bottles, such that for their complement each k -prism corresponds to a manifold with a $\mathbb{H}^2 \times \mathbb{R}$ metric of finite volume, and each almost Pogorelov polytope corresponds to a hyperbolic manifold of finite volume.*

Thus, for real moment-angle manifolds and small covers over simple 3-polytopes the following 5 Thurston’s geometries arise: S^3 for $P = \Delta^3$ (simplex), $S^2 \times \mathbb{R}$ for $P = \Delta^2 \times I$ (3-prism), $\mathbb{R}^3$ for $P = I^3$ (4-prism), $\mathbb{H}^2 \times \mathbb{R}$ for pieces corresponding to k -prisms, $k \geq 5$, $\mathbb{H}^3$ for pieces corresponding to almost Pogorelov polytopes different from k -prisms. In particular, if P is a k -prism, $k \geq 5$, then the manifold has a geometry of $\mathbb{H}^2 \times \mathbb{R}$, and if P is a *Pogorelov polytope*, that is an almost Pogorelov polytope without quadrangles, then the manifold is a compact hyperbolic manifold.

Key words and phrases: 3-manifold, Geometrization Conjecture of Thurston, prime manifold, JSJ-decomposition, small cover, almost Pogorelov polytope.

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The cohomology rings of smooth complex and real toric varieties

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Abstract: We phrase our results in the language of toric varieties. They equally hold for partial quotients of moment-angle complexes and real toric spaces.

Additively, the integral cohomology of a smooth toric variety $X_\Sigma(\mathbb{C})$ has been determined in [1]. The result is an isomorphism of graded $\mathbb{Z}$ -modules

$$H^*(X_\Sigma(\mathbb{C}); \mathbb{Z}) \cong \mathrm{Tor}_{H^*(BL; \mathbb{Z})}^*(\mathbb{Z}, \mathbb{Z}[\Sigma]) \quad (*)$$

where $\mathbb{Z}[\Sigma]$ is the Stanley–Reisner ring of Σ and L the torus acting on $X_\Sigma(\mathbb{C})$.

Contrary to a claim made by Buchstaber–Panov, the cup product in cohomology does not always correspond to the canonical product on the Tor term. It does so if one inverts 2, but in general there are twisting terms that we describe explicitly in terms of the fan Σ , see [3].

For coefficients in $\mathbb{Z}_2$, an additive isomorphism analogous to $(*)$ holds for the real locus of $X_\Sigma(\mathbb{C})$, which is the smooth real toric variety $X_\Sigma(\mathbb{R})$. This implies that smooth toric varieties are M-varieties. As before, the product is twisted, and we give an explicit formula for it [4].

In the complex case we also describe the morphism induced in cohomology by a toric morphism $X_\Sigma(\mathbb{C}) \rightarrow X_{\Sigma'}(\mathbb{C})$. Once again, it corresponds to the canonical map between the Tor terms if 2 is invertible, and to a deformation of it in general [5]. (This part is joint work with Xin Fu.)

The main technical tools of our theory are the notion of a homotopy Gerstenhaber algebra and the homotopy Gerstenhaber formality of Davis–Januszkiewicz spaces [2]. The latter generalizes the well-known dga formality result due to the speaker and Notbohm–Ray.

Key words and phrases: toric variety, partial quotient of moment-angle complex, real toric space, homotopy Gerstenhaber algebra

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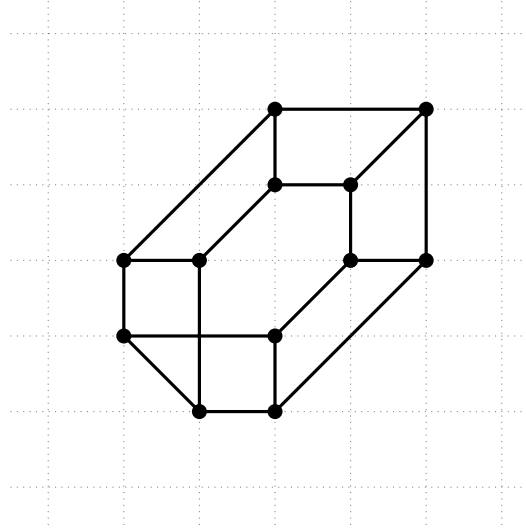
On the GKM correspondence in low dimensions

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Abstract: The GKM correspondence associates to a torus action of GKM type a labelled graph encoding the one-skeleton of the action. In the toric symplectic case, this graph is nothing but the one-skeleton of the momentum polytope. Two natural questions arise: in how far is the space acted on determined by the graph, and which graphs are realizable by GKM actions? We will discuss three low-dimensional (i.e., 6 and 8) results on these questions, obtained in joint work with Panagiotis Konstantis and Leopold Zoller:

- Every abstract 3-valent GKM graph that fibers over a 2-valent one is realized as the projectivization of an equivariant rank 2 vector bundle over a 4-dimensional T^2 -manifold. [2]
- The diffeomorphism type of a simply-connected, compact 6-dimensional integer GKM T^2 -manifold is encoded in the GKM graph. [1]
- In dimension 8, this is no longer true: we construct GKM T^3 -actions with identical GKM graph on two 8-dimensional compact manifolds with different homotopy type. [3]



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Universal simplicial complexes inspired by toric topology

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Abstract: Let $\mathbf{k}$ be the field $\mathbb{F}_p$ or the ring $\mathbb{Z}$. In this talk I'll discuss combinatorial and topological properties of the universal complexes $X(\mathbf{k}^n)$ and $K(\mathbf{k}^n)$ whose simplices are certain unimodular subsets of $\mathbf{k}^n$. I'll describe their $\mathbf{f}$ -vectors, show that they are shellable but not shifted, and mention their applications in toric topology and number theory.

As a main result I'll show that $X(\mathbf{k}^n)$, $K(\mathbf{k}^n)$ and the links of their simplices are homotopy equivalent to a wedge of spheres specifying the exact number of spheres in the corresponding wedge decompositions. This is a generalisation of Davis and Januszkiewicz's result that $K(\mathbb{Z}^n)$ and $K(\mathbb{F}_2^n)$ are $(n - 2)$ -connected simplicial complexes.

This is joint work with Djordje Baralić, Aleš Vavpetič, and Aleksandar Vučić.

Branched coverings of manifolds and finite transformation groups

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Abstract: In this talk I will focus on various problems concerning smooth manifolds X and their finite smooth transformation groups G such that the orbit space X/G is a smoothable topological manifold. Certainly, we consider non-free actions of finite groups. The case of an arbitrary finite transformation group is very difficult. Here I consider two special cases: (A) a finite number of commuting involutions, and (B) transformation groups of odd order.

Theorem α . (Transfer for group actions. Folklore) *Suppose X is a finite polyhedron, and G - a finite group acting on X simplicially. Then for any field $\mathbb{K}$ of characteristics zero or p , $(|G|, p) = 1$, the induced map $f^*: H^*(X/G; \mathbb{K}) \rightarrow H^*(X; \mathbb{K})$ is an isomorphism onto the G -invariant subring $H^*(X; \mathbb{K})^G$.*

There is also the well known variation of this theorem, where X is any paracompact (for us it will be enough to consider the case of metric compacts of finite dimension), and the singular cohomology is replaced by Čech cohomology. And, certainly, for ENR spaces Čech cohomology is canonically isomorphic to singular cohomology.

Consider a standard sphere $S^m = (x_1^2 + x_2^2 + \dots + x_{m+1}^2 = 1)$ (of nonzero dimension), and denote by $\tau(x_1, \dots, x_m, x_{m+1}) = (x_1, \dots, x_m, -x_{m+1})$ the standard involution of the sphere, that interchanges the Northern and the Southern Poles. Take a direct product of k spheres $M^m = S^{m_1} \times S^{m_2} \times \dots \times S^{m_k}$ of arbitrary dimensions. On each factor there acts the corresponding involution $\tau_i, 1 \leq i \leq k$. We get an action of k commuting involutions on M^m , and, therefore, the action of the group $G_k = \mathbb{Z}_2^k$. Let us consider the subgroup $\bar{G}_k \subset G_k$ of orientation preserving elements $g \in G_k$. This subgroup has index 2, so $\bar{G}_k$ is isomorphic to $\mathbb{Z}_2^{k-1}$. The following result was obtained in [4].

Theorem 1. (G., 2019) *In the above notation the orbit space $M^m/\bar{G}_k$ is the topological sphere S^m . Moreover, the projection $f: M^m \rightarrow M^m/\bar{G}_k$ can be realized by the following explicit formula:*

$$\begin{aligned} \chi: M^m &\rightarrow S^m, \\ \chi(x_{1,1}, \dots, x_{1,m_1}, x_{1,m_1+1}, x_{2,1}, \dots, x_{2,m_2}, x_{2,m_2+1}, \dots, x_{k,1}, \dots, x_{k,m_k}, x_{k,m_k+1}) &:= \\ &\frac{(x_{1,1}, \dots, x_{1,m_1}, x_{2,1}, \dots, x_{2,m_2}, \dots, x_{k,1}, \dots, x_{k,m_k}; x_{1,m_1+1} \cdot x_{2,m_2+1} \cdots x_{k,m_k+1})}{\sqrt{x_{1,1}^2 + \dots + x_{k,m_k}^2 + x_{1,m_1+1}^2 x_{2,m_2+1}^2 \cdots x_{k,m_k+1}^2}}. \end{aligned}$$

In addition, the branched covering $\chi: M^m \rightarrow S^m$ is globally C^ω -smooth and nondegenerate at the points of local homeomorphism.

It turns out that the number $(k - 1)$ of commuting involutions in this theorem is a sharp estimate. Recall, that the rational cup-length $L(X)$ of a connected ENR space X is the maximal integer m such that there exist m homogeneous cohomology classes $a_1, a_2, \dots, a_m \in H^*(X; \mathbb{Q})$ of positive degrees with nonzero product $a_1 a_2 \dots a_m \neq 0$.

Theorem 2. (G., 2020) *Let X be a smooth (PL, TOP) closed connected orientable manifold of dimension $m \geq 3$. If the rational cup-length $L(X)$ is equal to $k \geq 3$, then for any $(k-2)$ commuting involutions $\tau_1, \dots, \tau_{k-2}$ acting on X and preserving orientation, the orbit space $X/\mathbb{Z}_2^{k-2}$ has a nonzero rational Betti number $B_q(X/\mathbb{Z}_2^{k-2})$ (in Čech cohomology) for some $1 \leq q \leq m-1$. In particular, the orbit space $X/\mathbb{Z}_2^{k-2}$ is not a $\mathbb{Q}$ -homology sphere.*

Let us focus on the finite transformation groups of odd order. The following lemma is quite easy (modulo the celebrated Feit-Thompson theorem stating that any group of odd order is solvable), but I have not found it in literature.

Lemma 1. *Let G be a finite group of odd order, and $\mathbb{K} \subset \mathbb{R}$ be an arbitrary subfield. Then for any finite-dimensional linear space V over $\mathbb{K}$ and any linear representation G on V the following equation holds $\dim(V) = \dim(V^G) \pmod{2}$.*

Take a smooth oriented closed connected manifold X^m . Suppose it has some rational Betti number $B_q(X^m)$, $1 \leq q \leq m-1$, which is odd. Then by lemma 1 there is no action of any finite group G of odd order such that the orbit space X^m/G is a $\mathbb{Q}$ -homology sphere. For example, we have the following

Proposition 1. (G., 2020) *Consider an m -dimensional torus T^m . If m is not a power of 2, then there is no action of any finite group G of odd order such that the orbit space T^m/G is the sphere S^m .*

For T^2 there is a classical action of $\mathbb{Z}_3$ with the orbit space S^2 . For T^{2^k} , $k \geq 2$, the question is still open.

Let us denote by $w_i(Z)$, $1 \leq i \leq \dim(Z)$, the Stiefel-Whitney classes of a smoothable topological manifold Z .

Theorem 3. (G., 2020) *Let X be a smooth closed connected manifold of dimension $m \geq 3$, considered with a smooth effective action of a finite group G of odd order. Suppose the orbit space X/G is a smoothable topological manifold. Denote by $f: X \rightarrow X/G$ the canonical projection. Then the induced map in mod 2 cohomology $f^*: H^*(X/G; \mathbb{Z}_2) \rightarrow H^*(X; \mathbb{Z}_2)$ is an isomorphism onto the subring of invariants $H^*(X; \mathbb{Z}_2)^G$ and, moreover, $f^*(w_i(X/G)) = w_i(X)$, $1 \leq i \leq m$.*

In the paper [1] Alexander proved the classical

Theorem β . *Let X^m be a PL oriented closed connected manifold, $m \geq 2$. Then there exists a PL finite-fold branched covering $f: X^m \rightarrow S^m$.*

But in Alexander's construction the degree n of such a branched covering is always greater than $m!$. On the other side there is a classical lower bound for such an n , namely, $n \geq L(X^m)$. This is a particular case of the celebrated Berstein-Edmonds theorem (see [2]). A long standing question is that for any dimension m one can construct an n -fold PL branched covering $X^m \rightarrow S^m$ with $n = m$. It is true for $m = 2, 3, 4$, and is still open for $m \geq 5$ even for the torus T^m .

In the celebrated work of Chernavskii [3] it was obtained a good behavior properties for branched coverings of manifolds in TOP category.

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Key words and phrases: finite transformation groups, characteristic classes, branched coverings.

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Permutation bases for the cohomology rings of regular semisimple Hessenberg varieties

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Abstract: Recent work of Shareshian and Wachs, Brosnan and Chow, and Guay-Paquet connects the well-known Stanley-Stembridge conjecture in combinatorics to the dot action of the symmetric group on the cohomology rings $H^*(\text{Hess}(\mathbf{S}, h))$ of regular semisimple Hessenberg varieties. In particular, in order to prove the Stanley-Stembridge conjecture, it suffices to construct for any Hessenberg function h a permutation basis of $H^*(\text{Hess}(\mathbf{S}, h))$ whose elements have stabilizers isomorphic to reflection subgroups. In this talk I will outline several recent results which contribute to this goal. Specifically, in some special cases, we give a new, purely combinatorial construction of classes in $H^*(\text{Hess}(\mathbf{S}, h))$ which form permutation bases for subrepresentations in $H^*(\text{Hess}(\mathbf{S}, h))$. Our techniques use the Goresky-Kottwitz-MacPherson theory in equivariant cohomology. Special cases of our construction have appeared in past work of Abe-Horiguchi-Masuda, Timothy Chow, and Cho-Hong-Lee. This is a report on joint work with Martha Precup and Julianna Tymoczko.

Holomorphic foliations and transversal real submanifolds in Kähler manifolds

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Abstract: This talk is based on the ongoing joint work with Hisashi Kasuya (Osaka University). In this talk I will explain a new construction of a pair of a real submanifold and a holomorphic foliation in a Kähler manifold equipped with a torus action. This construction provides non-invariant complex structures on compact Lie groups and a generalization of LVM theory.

Key words and phrases: torus action, moment map, holomorphic foliation.

On homology of Lie algebras over commutative rings

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Abstract: In this talk I will tell you about five different types of homology of a Lie algebra over a commutative ring. These five types are naturally isomorphic over fields. However, they become non-isomorphic over commutative rings, even over $\mathbb{Z}$. We study connections between these types of homology, in particular, we construct some spectral sequences, that connect them. As an auxiliary result we prove that the Koszul complex of a module M over a principal ideal domain that connects the exterior and the symmetric powers $0 \rightarrow \Lambda^n M \rightarrow M \otimes \Lambda^{n-1} M \rightarrow \dots \rightarrow S^{n-1} M \otimes M \rightarrow S^n M \rightarrow 0$ is pure acyclic.

Key words and phrases: Homology, Lie algebras, derived functors, Chevalley-Eilenberg complex.

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Equivariant orbit preserving diffeomorphisms

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Abstract: Let a torus act smoothly on a manifold. Assume that the manifold is connected and the action is faithful. Then every orbit-preserving equivariant diffeomorphism is obtained from an invariant smooth map from the manifold to the torus by acting by the values of this function.

This theorem — in a more general setup of torus actions on orbifolds — appeared in Haefliger and Salem’s 1991 paper “Actions of tori on orbifolds”. The crucial step of the proof was attributed to Gerald Schwarz. Together with Gerald Schwarz, we close a gap in this proof.

Ring of conditions and cohomology ring of toric bundles

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Abstract: Schubert developed a special calculus in order to solve various problems on computation of intersection numbers of generic algebraic subvarieties of complementary dimensions in complex Grassmannians. In 1980-th De Concini and Procesi developed a theory of rings of conditions which generalized Schubert calculus for a wide class of not necessarily compact homogeneous spaces.

In the talk I will describe rings of conditions for so-called horospherical homogeneous spaces. This description relies on computation of cohomology rings of toric bundles, i.e. of smooth equivariant compactifications of torus principal bundles over a smooth (not necessarily algebraic) compact base.

The talk is based on joint paper with Johannes Hofscheier and Leonid Monin.

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Two-dimensional Golod complexes

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Abstract: Although Golodness of a triangulation of a closed surface is characterized by its neighborliness through the fat-wedge filtration of a polyhedral product as in [1], Golodness of a general two-dimensional simplicial complex is not characterized by its neighborliness. I will talk about the characterization of Golodness of a general two-dimensional simplicial complex by introducing a new notion called *vertex-breakability*. Our characterization applies to give a simple example of a Golod complex whose moment-angle complex is not a suspension.

This is joint work with Kouyemon Iriye [2].

Key words and phrases: Stanley-Reisner ring, Golod property, neighborly complex, polyhedral product, fat-wedge filtration.

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Khovanov homology via immersed curves

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Abstract:

Khovanov homology is a powerful combinatorially defined knot invariant. We will describe a novel geometric approach to Khovanov homology, where the topological input is a two-sphere S intersecting a knot K in 4 points. The geometric outcome is that the dimension of Khovanov homology $\dim[Kh(K)]$ is equal to the minimal intersection number of a pair of specifically constructed immersed curves on the dividing 4-punctured sphere S . We will also discuss applications of this result to tangle replacement questions, such as mutation invariance of $Kh(K; Q)$ and the Cosmetic Crossing Conjecture. This is joint work with Liam Watson and Claudius Zibrowius.

Gale dual of a GKM graph and its application to the extension problem of torus actions

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Abstract: Let (Γ, α, ∇) be a GKM graph with an axial function $\alpha : E \rightarrow H^2(BT^n)$, where E is the set of edges of Γ . In the paper [1], we define the group of axial functions $\mathcal{A}(\Gamma, \alpha, \nabla)$ which is isomorphic to the finite rank abelian group without torsions. We have proved that if $\mathcal{A}(\Gamma, \alpha, \nabla) \simeq \mathbb{Z}^\ell$, then there exists an extension $\tilde{\alpha} : E \rightarrow H^2(BT^\ell)$ of the axial function $\alpha : E \rightarrow H^2(BT^n)$ such that $(\Gamma, \tilde{\alpha}, \nabla)$ is a GKM graph; moreover, this is the maximal extension among axial functions α' such that $(\Gamma, \alpha', \nabla)$ becomes a GKM graph. However, in general, to compute the rank of $\mathcal{A}(\Gamma, \alpha, \nabla)$ is complicated because we need to make and solve many (at most number of edges) systems of equations.

In this talk I will introduce the Gale dual (Γ, ρ, ∇) of the complexity one GKM graph (Γ, α, ∇) , i.e., Γ is an $(n+1)$ -valent graph and the axial function is $\alpha : E \rightarrow H^2(BT^n)$. Here the label $\rho : E \rightarrow \mathbb{Z}$ is defined by

$$\rho(e_1)\alpha(e_1) + \cdots + \rho(e_{n+1})\alpha(e_{n+1}) = 0$$

for every vertex p and its out going edges $\{e_1, \dots, e_{n+1}\} (= E_p)$. Then, I will explain the following theorem by using examples:

Theorem. Let (Γ, α, ∇) be an $(n+1)$ -valent complexity one GKM graph and (Γ, ρ, ∇) be its Gale dual. Then, $\mathcal{A}(\Gamma, \alpha, \nabla) \simeq \mathbb{Z}^{n+1}$ if and only if $(\rho(e_1), \dots, \rho(e_{n+1})) \in \mathcal{A}(\Gamma, \alpha, \nabla)_p$ for some $p \in V$ and $\{e_1, \dots, e_{n+1}\} (= E_p)$.

Here $\mathcal{A}(\Gamma, \alpha, \nabla)_p$ in Theorem is the restriction of the elements of group of axial functions $\mathcal{A}(\Gamma, \alpha, \nabla)$ onto the vertex p .

This is a joint work with Tomoo Matsumura (see the paper [2]).

Key words and phrases: Extension problem of torus actions, GKM graph, Gale dual.

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Infinite transitivity of regular automorphisms on Calogero-Moser spaces

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Abstract: Consider an algebraic variety X and the group of all its regular automorphisms. For which X any m its (smooth) points can be moved to any other m (smooth) points via a regular automorphism? Such varieties are quite rare. In the work [3] by Berest-Eshmatov-Eshmatov the simplest Calogero-Moser space $\mathcal{C}_n$ was studied from this point of view. This variety holds an action of the group isomorphic to the unimodular automorphism group of the free associative algebra on two variables, this is a subgroup of its regular automorphism group. Berest-Eshmatov-Eshmatov proved 2-transitivity of this action and conjectured that this action is infinitely transitive. In the talk, we will discuss the anatomy of $\mathcal{C}_n$ and sketch the proof of infinite transitivity.

For details on infinitely transitive actions, see [1] and [2]. Details of the proof of infinite transitivity for Calogero-Moser spaces can be found in [4].

Key words and phrases: Calogero-Moser space, infinitely transitive action, automorphism group.

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On the torus orbit closures in flag varieties

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Abstract: Let G be a semisimple Lie group over $\mathbb{C}$, B a Borel subgroup of G , and T a maximal torus of G contained in B . The left multiplication of T on G induces an action of T on the flag variety G/B . It has been known that the closure of a *generic* torus orbit in G/B , which means that the closure contains all the T -fixed points in G/B , is a smooth projective toric variety whose fan is associated with the Weyl chamber. In this talk, we consider a *not* necessarily generic torus orbit closure in G/B , and we study how to describe the fan of a torus orbit closure using the matroid maps, which we call *retractions*. This talk is based on joint work with Mikiya Masuda and Seonjeong Park.

Key words and phrases: flag varieties, toric varieties, Coxeter matroids.

On polyhedral products over minimally non-Golod complexes

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Abstract: A simplicial complex is called minimally non-Golod if it is not Golod itself (that is, Tor-algebra of its face ring over a certain field has a non-trivial product or Massey bracket), but removing of any vertex yields a Golod complex. The results obtained recently by Amelotte and by Grbic, Ilyasova, Panov, Simmons show that this property of a complex is closely related to those of its (real) moment-angle-complex (being a connected sum of products of spheres with two spheres in each product), its Pontryagin algebra (being a one-relator algebra), and the commutator subgroup of its Right-Angled Coxeter group (being a one-relator group), when the simplicial complex is flag.

In this talk I will recall their results, point out that a similar relationship is likely to exist in a wide class of non-flag minimally non-Golod triangulated spheres, and finally introduce some examples showing that non of the above mentioned properties hold for a general minimally non-Golod complex.

On simple orbifolds

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Abstract: In this talk I will give the notion of simple orbifolds. We will pay more attention on a special class of simple orbifolds, called simple handlebodies, each of which is a simple orbifold whose underlying spaces is a handlebody (i.e., the tubular neighborhood of the wedge of some circles in $\mathbb{R}^n$) and can be cut into a simple polytope along some specific codimension-one suborbifolds. We generalize the notions of flag and belt in the setting of simple polytopes to the setting of simple handlesbodies, with a difference. Furthermore, we study the relations among topology, geometry, combinatorics and the orbifold fundamental groups of simple handlebodies. This is a joint work with Lisu Wu.

Key words and phrases: simple handlebody, flagness, asphericity, orbifold fundamental group.

Invariants of the cohomology rings of the permutohedral varieties

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Abstract: The permutohedral variety X of complex dimension $n - 1$ is a smooth compact toric variety associated to the permutohedron of dimension $n - 1$. The natural action of the symmetric group $\mathfrak{S}_n$ on X induces the action on the cohomology $H^*(X; \mathbb{Q})$ and the ring structure of its $\mathfrak{S}_n$ -invariants $H^*(X; \mathbb{Q})^{\mathfrak{S}_n}$ is explicitly described by Klyachko [1]. In this talk I will discuss the ring structure of the invariants $H^*(X; \mathbb{Q})^{\mathfrak{S}_\lambda}$ under the restricted action of a Young subgroup $\mathfrak{S}_\lambda$. This talk is based on joint work with T. Horiguchi, J. Shareshian and J. Song.

Key words and phrases: permutohedral variety, Young subgroup, invariants.

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Polyhedral products, loop homology, and right-angled Coxeter groups

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Abstract: Using results on the topology of polyhedral products, we link distinct concepts of homotopy theory and geometric group theory. On the homotopical side, we describe the Pontryagin algebra (loop homology) of the moment-angle complex $\mathcal{Z}_K$. On the group-theoretical side, we describe the structure of the commutator subgroup RC'_K of a right-angled Coxeter group RC_K , viewed as the fundamental group of the real moment-angle complex $\mathcal{R}_K$. For a flag simplicial complex K , we present a minimal generator set for the Pontryagin algebra $H_*(\Omega\mathcal{Z}_K)$ and for the commutator subgroup RC'_K , and specify a necessary and sufficient combinatorial condition for $H_*(\Omega\mathcal{Z}_K)$ and RC'_K to be a free or one-relator algebra (group). We also give homological characterisations of these properties. For RC'_K , this is given by a condition on the homology group $H_2(\mathcal{R}_K)$, whereas for $H_*(\Omega\mathcal{Z}_K)$ this can be stated using the bigrading of the homology of $\mathcal{Z}_K$.

Parts of this talk are joint works with Jelena Grbic, Marina Ilyasova, George Simmons, Stephen Theriault, Yakov Veryovkin and Jie Wu.

Uniqueness of a toric structure on a Fano Bott manifold

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Abstract: A smooth Fano variety is a smooth projective variety X whose anticanonical divisor $-K_X$ is ample. In this talk, we consider the conjecture that two smooth Fano toric varieties are isomorphic if there exists a c_1 -preserving isomorphism between their integral cohomology rings. This conjecture is related to McDuff's question which asks whether a smooth Fano toric variety may have more than one toric structure. We show that the conjecture above is true for Fano Bott manifolds. This implies that a toric structure on a Fano Bott manifold is unique. This is a joint work with Yunhyung Cho, Eunjeong Lee, and Mikiya Masuda.

Details can be found in [1].

Key words and phrases: Bott manifold, Fano variety, toric action.

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Automorphisms of affine surfaces, the Markov numbers, and the Thompson group T

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Abstract: We describe the correspondence between the unit circle and the divisor at infinity for some affine algebraic surfaces, which induces the correspondence between the Thompson group T and the automorphism groups of considered surfaces.

In particular, we generalise the Markov surface and describe automorphism groups of the obtained surfaces. Finally, we raise the question about integer point orbits on these surfaces. The answer to it might be useful in studying the Uniqueness conjecture for Markov numbers.

Details can be found in [1] for the Markov numbers and the Uniqueness conjecture, in [3] and [4] for the Thompson group T , and in [2] for the surface transformations that we use.

Key words and phrases: affine surface, automorphism group, boundary divisor, Markov number, Thompson group.

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Some topological properties of monotone complexity one spaces

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Abstract:

In symplectic geometry it is often the case that compact symplectic manifolds with large group symmetries admit indeed a Kähler structure. For instance, if the manifold is of dimension $2n$ and it is acted on effectively by a compact torus of dimension n in a Hamiltonian way (namely, there exists a moment map which describes the action), then it is well-known that there exists an invariant Kähler structure. These spaces are also called complexity-zero spaces, where the complexity is given by n minus the dimension of the torus. In this talk I will explain how there is some evidence that a similar statement holds true when the complexity is one and the manifold is monotone (the latter being the symplectic analog of the Fano condition in algebraic geometry), namely, that every monotone complexity-one space is simply connected and has Todd genus one, properties which are also enjoyed by Fano varieties.

These results were largely inspired by a conjecture posed by Fine and Panov in 2010, by work of Lindsay and Panov of 2019, and it is joint with Isabelle Charton and Daniele Sepe.

Some of the details can be found in [1].

Key words and phrases: torus action, Hamiltonian spaces, complexity one, monotone.

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On LS-category and topological complexity of some fiber bundles and Dold manifolds

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Abstract: In this talk, I'll extend the product formula for Lusternik–Schnirelmann category and topological complexity to some classes of fiber bundles. Then I'll compute these invariants for Dold manifolds. This is a joint work with B. Naskar.

On the maximal extension problem for GKM-graphs

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Abstract: In 1998, M. Goresky, R. Kottwitz and R. MacPherson [1] introduced a special class of $(S^1)^k$ -actions on any smooth manifold M^{2n} with vanishing odd cohomology. They computed the respective equivariant cohomology ring in terms of so-called GKM-graphs. Roughly speaking, an (n, k) -type GKM-graph is an n -regular connected graph with $\mathbb{Z}^k$ -labels on its edges. In 2001, V. Guillemin and C. Zara [2] introduced GKM-graphs as independent combinatorial objects and studied the associated ring. GKM fiber bundles were introduced by V. Guillemin, S. Sabatini and C. Zara [3] in 2012. This subclass of GKM-graphs consists of combinatorial GKM-analogues for torus-equivariant fiber bundles.

In 2019, S. Kuroki [4] defined the free abelian group for any GKM-graph and called it the group of axial functions. He showed that this group is isomorphic to the group generated by the edge labels of the maximal extension for a given GKM-graph. In particular, for a given GKM-manifold the rank of the respective axial function group is an upper bound for the dimensions of the tori that appear in the extensions of the given GKM-action. The explicit computation of the axial function group is an open problem. It was recently conjectured that the group of axial functions has rank n for any 4-linearly independent n -regular GKM-graph.

In the talk (joint work with S. Kuroki) for any n -regular GKM fiber bundle (satisfying some mild conditions) we give the criterion for the axial function group to have rank n . In addition, we disprove the above conjecture by presenting a new example of an $(n + 1)$ -linearly independent $(n + 1 + r, n + 1)$ -type GKM-graph with axial function group of rank $n + 1$ for any integers $n > 1, r > 0$.

Key words and phrases: torus action, GKM-graph, GKM fiber bundle.

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Homotopy classification of 4-dimensional toric orbifolds

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Abstract: A toric orbifold [1] is a $2n$ -dimensional compact orbifold equipped with a locally standard T^n -action whose orbit space is a simple polytope. It generalizes the category of (quasi)toric manifolds to the orbifold category. This talk aims to study certain CW-complexes that model 4-dimensional toric orbifolds and investigate their homotopy theory. We show that a 4-dimensional toric orbifold without 2-torsion in its cohomology is homotopy equivalent to the wedge of a Moore space and a CW-complex consisting of even-dimensional cells. As a corollary, we prove that the integral cohomology ring distinguishes the homotopy type of such orbifolds. This is a joint work with Xin Fu and Tseleung So.

Key words and phrases: toric orbifold, homotopy type, cohomological rigidity.

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Realization of cohomology modulo torsion.

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Abstract: A classical problem in homotopy theory asks which graded rings are the cohomology of a space. We simplify this problem by asking which graded rings are the cohomology modulo torsion of a space. We use polyhedral products to show that any graded symmetric algebra over $\mathbb{Z}$, modulo a monomial ideal is the cohomology modulo torsion of a space.

This is joint work with Larry So.

The universal space of parameters for T^n -action on the Grassmannians $G_{n,2}$

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Abstract: It is a classical problem to study the structure of the standard action of the compact torus T^n on the Grassmann manifold $G_{n,2}$. This problem is closely related with series of problems in algebraic topology, algebraic geometry and mathematical physics, as it will be demonstrated in this talk as well.

We recently constructed the model for $(G_{n,2}, T^n)$ that is a smooth manifold with corners $U_n = \Delta_{n,2} \times \mathcal{F}_n$ and the continuous surjection $G : U_n \rightarrow G_{n,2}/T^n$, which implies that the orbit space $G_{n,2}$ is homeomorphic to the quotient space of U_n by the map G . Here $\Delta_{n,2}$ is the hypersimplex which in this context appears as an image of the standard moment map $\mu : G_{n,2} \rightarrow \mathbb{R}^n$ and $\mathcal{F}_n$ is a new smooth compact manifold. This manifold we call the universal space of parameters.

In the talk we explain our explicit construction of the universal space of parameters $\mathcal{F}_n$. This construction essentially uses the techniques of wonderful compactification coming from algebraic geometry and singularity theory. Moreover, we show that our universal space of parameters coincide with the Deligne-Mumford compactification of the moduli space of n - pointed Riemann sphere $\mathbb{CP}^1$.

The talk is based on joint work with Victor M. Buchstaber.

Key words and phrases: torus action, Grassmann manifolds, universal space of parameters.

The homotopy type of the based loops on a moment-angle complex

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Abstract: An interesting problem is to determine the homotopy type of a moment-angle complex. An appealing application of this is given by Buchstaber and Panov, who showed that the problem is equivalent to determining the homotopy type of the complement of a corresponding complex coordinate subspace arrangement. A variety of cases have been worked out but the problem gets difficult quickly. In this talk motivation is given as to why it is also interesting to consider the homotopy type of the based loops on a moment-angle complex, and show that methods from homotopy theory (with no immediate connection to toric topology) can be used to work out additional cases.

Four-dimensional Generalized-Kähler structures and T-duality

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Abstract: A hyperkahler manifold is a Riemannian manifold of dimension $4k$ and holonomy group contained in $\mathrm{Sp}(k)$. Such manifolds are always Kahler and Ricci-flat and thus are examples of Calabi-Yau manifolds. Complete asymptotically flat hyperkahler 4-manifolds are also known as gravitational instantons and are of a special interest in physics. In 1978, Gibbons and Hawking suggested an ansatz allowing to construct an infinite family of complete asymptotically flat Hyperkahler 4-manifolds *with S^1 -symmetry*, unifying a large zoo of examples known before. The presence of S^1 -symmetry is crucial for this ansatz, as it allows to reduce a complicated nonlinear system of PDEs to a single Laplace equation.

In this talk we explain how this construction can be modified to produce a much more flexible geometric structure of a Generalized Kähler (GK) manifold. By definition, a GK manifold carries two integrable complex structures I and J and a (not necessarily Kähler) bi-Hermitian metric, satisfying certain compatibility conditions. We provide an exhaustive local ansatz for GK 4-manifolds with S^1 -symmetry. As an essential part of our construction, we will use the operation of T-duality, which provides an involution on the set of all GK structures with a free S^1 -action.

This talk is based on a joint project with Jeffrey Streets and more details can be found in [1].

Key words and phrases: GK structures, S^1 -symmetry, Gibbons-Hawking ansatz, moment map.

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[1] J. Streets, Y. Ustinovskiy *The Gibbons-Hawking ansatz in generalized Kähler geometry*, preprint arXiv:2009.00778

Polyhedral products and commutator subgroups of right-angled Artin and Coxeter groups

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Abstract: We construct and study polyhedral product models for classifying spaces of right-angled Artin and Coxeter groups, general graph product groups and their commutator

subgroups. By way of application, we give a criterion of freeness for the commutator subgroup of a graph product group, and provide an explicit minimal set of generators for the commutator subgroup of a right-angled Coxeter group.

This is a joint work with Taras Evgenievich Panov.

The author is the winner of the 2019 “Young mathematics competition in Russia”.

This work is supported by the Russian Science Foundation under grant 20-11-19998.

Details can be found in the work [1].

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Taras Panov, Yakov Veryovkin, *Polyhedral products and commutator subgroups of right-angled Artin and Coxeter groups*, Russian Academy of Sciences Sbornik Mathematics 207(11). DOI: 10.1070/SM8701.

A combinatorial approach to the exponents of Moore spaces

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Abstract: This talk gives a report on the combinatorial approach to the exponents of Moore spaces using Cohen’s program. By working out the combinatorics of the Cohen groups, we obtain that the power map p^{r+1} on the loop space of the mod p^r Moore space $P^{2n+1}(p^r)$ projecting to the atomic piece $T^{2n+1}\{p^r\}$ is null homotopic for $p > 3$, $n > 1$ and $r > 1$. This gives a shorter proof on the exponents of Moore spaces using the Cohen groups. This is a joint work with Fred Cohen and Roman Mikhailov.

Details can be found in the work [1]. This work is one of our efforts for attacking the Barratt conjecture on the exponents of homotopy groups. Some other recent works on this topic can be found in [2,3]. The detailed descriptions on the combinatorial approach to the homotopy theory of loop suspensions can be found in the books [4,5].

Key words and phrases: homotopy groups, Moore spaces, Cohen groups.

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On the Picard groups of regular semisimple Hessenberg varieties

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Abstract: Regular semisimple Hessenberg varieties are a family of smooth subvarieties of the full flag varieties G/B . This family contains the full flag variety and Permutohedral variety. In this talk we will discuss on the Picard groups of regular semisimple Hessenberg varieties. This is joint work with Hiraku Abe and Naoki Fujita in progress.

Key words and phrases: second Betti number, Permutohedral varieties, Hessenberg functions.

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