NATIONAL RESEARCH UNIVERSITY

Conference

New Frontiers in High-Dimensional Probability and Applications to Machine Learning

Venue: Sirius University of Science and Technology
(Sochi, Olympiisky prospect 1)

May 12, 2021

Time	Speaker	Title
7:00 – 11:00	Breakfast (Omega Sirius)	
14:30 – 15:00	Registration (Sirius University)	
15:00 – 15:30	Welcome Coffee Break	
15:30 – 16:15	Dmitry Zaporozhets (PDMI RAS)	Gaussian from its maximum
16:15 –16:45	Alexey Naumov (HSE University)	Rates of convergence for density estimation with GANs
16:45 – 17:15	Viacheslav Borovitskiy (St.Petersburg State University)	Gaussian process priors for functions with non-Euclidean domain
17:15 – 17:45	Tikhon Bogachev (HSE University, Huawei MRC)	Mathematical Problems of Optimal Scheduling
19:00 – 21:00	Dinner (Omega Sirius)	

May 13, 2021

Time	Speaker	Title
7:00 – 10:00	Breakfast (Omega Sirius)	
10:00 – 11:00	Vladimir Bogachev (MSU, HSE University) – lecture 1	Spaces of measures: topology, geometry and low-dimensional approximations
11:00 – 11:15	Coffee Break	
11:15 – 12:00	Mikhail Lifshits (St.Petersburg State University)	Brownian chain break and some exit problems

Time	Speaker	Title
12:00 – 12:50	Evgeny Stepanov (HSE University, PDMI RAS)	Learning manifold structure from intrinsic distances: from multidimensional scaling to semidefinite programming
12:50 – 14:30	Lunch (Omega Sirius)	
14:30 – 15:30	Sergey Bobkov (University of Minnesota, HSE University) – lecture 1	Review of some results on limit theorems for densities
15:30 – 15:45	Coffee Break	
15:45 – 16:30	Quentin Paris (HSE University)	Online learning with exponential weights in metric spaces
16:30 – 17:15	Alexander Tikhomirov (Komi SC RAS)	On the local Marchenko-Pastor law for sparse sample covariance matrices
17:15 – 17:45	Sergey Samsonov (HSE University)	On the Stability of Random Matrix Product with Markovian Noise: Application to Linear Stochastic Approximation and TD Learning
17:45 – 18:15	Alexander Kalinin (MSU)	The extreme properties of concave measures and localization method
19:00 – 21:00	Dinner (Omega Sirius)	

May 14, 2021

7:00 – 11:00	Breakfast (Omega Sirius)
Leisure Time	

May 15, 2021

Time	Speaker	Title
7:00 – 10:00	Breakfast (Omega Sirius)	
10:00 – 11:00	Sergey Bobkov (University of Minnesota, HSE University) – lecture 2	Review of some results on limit theorems for densities
11:00 – 11:15	Coffee Break	

Time	Speaker	Title
11:15 – 12:00	Stanislav Shaposhnikov (MSU)	On uniqueness of probability solutions to the Fokker-Planck-Kolmogorov equation
12:00 – 12:50	Alexander Kolesnikov (HSE University)	On the functional Blaschke-Santalo inequality and related optimal transportation problem
12:50 – 14:30	Lunch (Omega Sirius)	
14:30 – 15:30	Vladimir Bogachev (MSU, HSE University) – lecture 2	Spaces of measures: topology, geometry and low-dimensional approximations
15:30 – 15:45	Coffee Break	
15:45 – 16:15	Egor Kosov (MSU, HSE University)	Total variation distance bounds for distributions of polynomials in Gaussian random variables
16:15 – 16:45	Alexander Zimin (HSE University)	Methods of Optimal Transportation in optimal multi-good auctions design
16:45 – 17:15	Nikolay Tolmachev (MSU)	TBA
17:15 – 17:45	Daniil Tiapkin (HSE University)	On Complexity Bounds of the Wasserstein Barycenter Problem
19:00 – 21:00	Conference Dinner (Omega Sirius)	

Abstracts

Mini-courses:

Vladimir Bogachev (MSU, HSE University)

Spaces of measures: topology, geometry and low-dimensional approximations

We discuss basic properties of spaces of measures on topological or metric spaces with their natural topologies (weak and setwise). Various subsets of the whole space of bounded signed measures will be considered, including the set of probability measures. Metrizability and compactness will be discussed, in particular, Kantorovich and Prohorov type metrics. Several problems connected with approximations by measures with low-dimensional supports will be studied.

Sergey Bobkov (University of Minnesota, HSE University)

Review of some results on limit theorems for densities

We will review several results in the central limit theorem on the convergence to the normal distributions. In particular, classical theorems due to Gnedenko (for uniform distance), Prokhorov (convergence in L^1), and Barron (the entropic CLT) will be presented as particular cases of a more general statement in Orlicz spaces using a unifying decomposition approach.

Talks:

Tikhon Bogachev (HSE University, Huawei MRC)

Mathematical Problems of Optimal Scheduling

Optimal distribution of effort is essential whenever we deal with a certain amount of jobs and the processing resource we have is limited. In particular, it happens inside communication networks and takes the form of scheduling of queueing systems. In this research a fluid model for resource allocation is considered and a local prediction on the system behavior is developed. As a result we obtain a set of possible cases, some of which lead to quite clear multidimensional problems.

Viacheslav Borovitskiy (St.Petersburg State University)

Gaussian process priors for functions with non-Euclidean domain

Gaussian processes are widely used in machine learning as priors for nonparametric Bayesian inference. Methods based on Gaussian processes are often considered a gold standard for scenarios where well-calibrated predictive uncertainty is of utter importance, such as decision making. It is important for applications to have a class of "general purpose" Gaussian process

priors. Traditionally, Matérn Gaussian processes serve as such a class to model functions with Euclidean domain, but for functions with non-Euclidean domains there is no such class. This state of affairs hinders the use of Gaussian process based methods for a number of applications such as robotics and optimization. In this talk I will describe a way to extend the definition of Matérn Gaussian processes to Riemannian manifolds and undirected weighted graphs, and then talk about practical ways to use them. I will also briefly discuss how these models can help solve practical problems.

The talk will be based upon the following two papers of the speaker:
Matérn Gaussian Processes on Riemannian Manifolds (NeurIPS 2020)
Matérn Gaussian Processes on Riemannian Graphs (AISTATS 2021)

Dmitry Zaporozhets (PDMI RAS)

Gaussian from its maximum

In this talk, we will discuss the following question. Suppose that
\[
X=(X_1,\dots,X_n)
\]
is a centered Gaussian vector in $\mathbb R^n$. Is it true that the
distribution of the
\[
\max_{k=1,\dots,n}(X_1,\dots,X_n)
\]
uniquely (up to permutations) defines the distribution of X?

The talk is based on a joint work with Maria Dospolova.

Alexander Zimin (HSE University)

Methods of Optimal Transportation in optimal multi-good auctions design

We will discuss the optimal auction design and consider its generalization for the case of multiple goods. We will talk about the possible applications of the optimal transportation theory, consider the dual approach and look at the numerical simulations based on it.

Alexander Kalinin (MSU)

The extreme properties of concave measures and localization method

We consider extreme points of collections of concave measures on infinite dimensional locally convex spaces and some techniques for localization measures. We describe the most important properties for collections of measures with finite number of integral conditions. A lot of measures can be localized from whole space to smaller subspace or small convex set with the same integral conditions. There is a localization method for concave measures on infinite dimensional locally

convex spaces with only one integral condition. The main result is a nontrivial modification of this method in case of any finite number of integral conditions and geometrical description of its results. By this algorithm we can construct the special concave measure concentrated on an infinite dimensional convex set and satisfy all integral conditions of the original measure which will be obtained as the extreme point of some collection of convex measures.

Alexander Kolesnikov (HSE University)

On the functional Blaschke-Santalo inequality and related optimal transportation problem

Motivated by the geodesic barycenter problem from optimal transportation theory, we prove a natural generalization of the Blaschke-Santalo inequality for many sets and many functions. We derive from it an entropy bound for the total Kantorovich cost appearing in the barycenter problem. The talk is based on joint work with Elisabeth Werner.

Egor Kosov (MSU, HSE University)

Total variation distance bounds for distributions of polynomials in Gaussian random variables

In the talk we discuss the recent results concerning upper bounds for the total variation distance between two distributions of polynomials in Gaussian random variables in terms of L^2 distance between polynomials and Kantorovich–Rubinstein distance between distributions. These bounds are often connected to the regularity properties of distributions, so we also present the recent results on this topic.We also discuss possible improvements of the general results for polynomials of a special type, e.g. for second degree polynomials.

Mikhail Lifshits (St.Petersburg State University)

Brownian chain break and some exit problems

Interacting Brownian particles are a popular model for various physical systems where a number of particles is subjected to inter-particle forces and ambient noise. We investigate in depth the behaviour of one such model studied earlier both by physicists and mathematicians: a finite chain of Brownian particles, interacting through a pairwise quadratic potential, with one end of the chain fixed and the other end pulled away at slow speed. We study the instant when the chain "breaks", that is, the distance between two neighboring particles becomes larger than a certain limit. There are three different regimes depending on the relation between the speed of pulling and the Brownian noise. We prove weak limit theorems for the break time and the break position for each regime. The main tools used are weak dependence (Kolmogorov-Rozanov mixing theorem or Berman condition) and Piterbarg-Pickands theorem on Gaussian large deviations.
As a byproduct, for a class of Gaussian stationary processes we obtain a limit theorem for the last exit time over a slowly growing linear boundary.

Quentin Paris (HSE University)

Online learning with exponential weights in metric spaces

This paper addresses the problem of online learning in metric spaces using exponential weights. We extend the analysis of the exponentially weighted average forecaster, traditionally studied in a Euclidean settings, to a more abstract framework. Our results rely on the notion of barycenters, a suitable version of Jensen's inequality and a synthetic notion of lower curvature bound in metric spaces known as the measure contraction property. We also adapt the online-to-batch conversion principle to apply our results to a statistical learning framework.

Alexey Naumov (HSE University)

R*ates of convergence for density estimation with GANs*

We undertake a precise study of the non-asymptotic properties of vanilla generative adversarial networks (GANs) and derive theoretical guarantees in the problem of estimating an unknown d-dimensional density p under a proper choice of the class of generators and discriminators. We prove that the resulting density estimate converges to p in terms of Jensen-Shannon (JS) divergence at the rate $n^{-2\beta/(2\beta+d)}$ where n is the sample size and β determines the smoothness of p. This is the first result in the literature on density estimation using vanilla GANs with JS rates faster than $n^{-1/2}$ in the regime $\beta>d/2$.

Sergey Samsonov (HSE University)

On the Stability of Random Matrix Product with Markovian Noise: Application to Linear Stochastic Approximation and TD Learning

This paper studies the exponential stability of random matrix products driven by a general (possibly unbounded) state space Markov chain. It is a cornerstone in the analysis of stochastic algorithms in machine learning (e.g. for parameter tracking in online-learning or reinforcement learning). The existing results impose strong conditions such as uniform boundedness of the matrix-valued functions and uniform ergodicity of the Markov chains. Our main contribution is an exponential stability result for the p-th moment of random matrix product, provided that (i) the underlying Markov chain satisfies a super-Lyapunov drift condition, (ii) the growth of the matrix-valued functions is controlled by an appropriately defined function (related to the drift condition). Using this result, we give finite-time p-th moment bounds for constant and decreasing stepsize linear stochastic approximation schemes with Markovian noise on general state space. We illustrate these findings for linear value-function estimation in reinforcement learning. We provide finite-time p-th moment bound for various members of temporal difference (TD) family of algorithms.

Evgeny Stepanov (HSE University, PDMI RAS)

Learning manifold structure from intrinsic distances: from multidimensional scaling to semidefinite programming

We will consider one of the important classes of manifold learning problems frequently arizing in applications of statistical data analysis: reconstruct the unknown manifold or its embedding into a given (say, Euclidean) space (or at least some of its structural characteristics) knowing just the information on intrinsic distances between points in its "almost dense'' subset. One of the basic spectral methods to solve such problems is multidimensional scaling (MDS). We will study whether the manifold is reconstructed by MDS (and what is actually reconstructed), and provide some negative though notrivial answers to this question. As an alternative a variational reconstruction methods based on semidefinite programming will be proposed.

Alexander Tikhomirov (Komi SC RAS)

On the local Marchenko-Pastor law for sparse sample covariance matrices

The present talk is based on the results recently obtained jointly with F. Göttze and D. Timushev. We consider sparse rectangular random matrices with sparsity probability p goes to zero as reciprocal dimension of matrix with some logarithmic factor. We assume that the entries of matrices are truncated on the level growing as the mean number of non-zero entries in the rows of the matrix. We give a full analysis of asymptotic behavior of the Stieltjes transform of spectral distribution function of sparse rectangular matrices. We describe as well the behaviour of eigenvalues of sample covariance matrices. Typical example of such matrices is the incidence matrix bipartite random graphs or lost data matrices.

Daniil Tiapkin (HSE University)

On Complexity Bounds of the Wasserstein Barycenter Problem

The talk is based on joint work with Darina Dvinskikh "Improved Complexity Bounds in Wasserstein Barycenter Problem". In this paper, we focus on computational aspects of the Wasserstein barycenter problem. We propose two algorithms to compute Wasserstein barycenters of m discrete measures of size n with accuracy ϵ. The first algorithm, based on mirror prox with a specific norm, meets the complexity of celebrated accelerated iterative Bregman projections (IBP), namely $\widetilde O(mn^2\sqrt n/\epsilon)$, however, with no limitations in contrast to the (accelerated) IBP, which is numerically unstable under small regularization parameter. The second algorithm, based on area-convexity and dual extrapolation, improves the previously best-known convergence rates for the Wasserstein barycenter problem enjoying $\widetilde O(mn^2/\epsilon)$ complexity.

Stanislav Shaposhnikov (MSU)

On the uniqueness of probability solutions to the Fokker-Planck-Kolmogorov equation

We will discuss new uniqueness results for solutions to Fokker-Planck-Kolmogorov equations for probability measures on finite and infinite-dimensional spaces. Moreover the relationship between uniqueness problems and the superposition principle will be considered.

Nikolay Tolmachev (MSU)

TBA