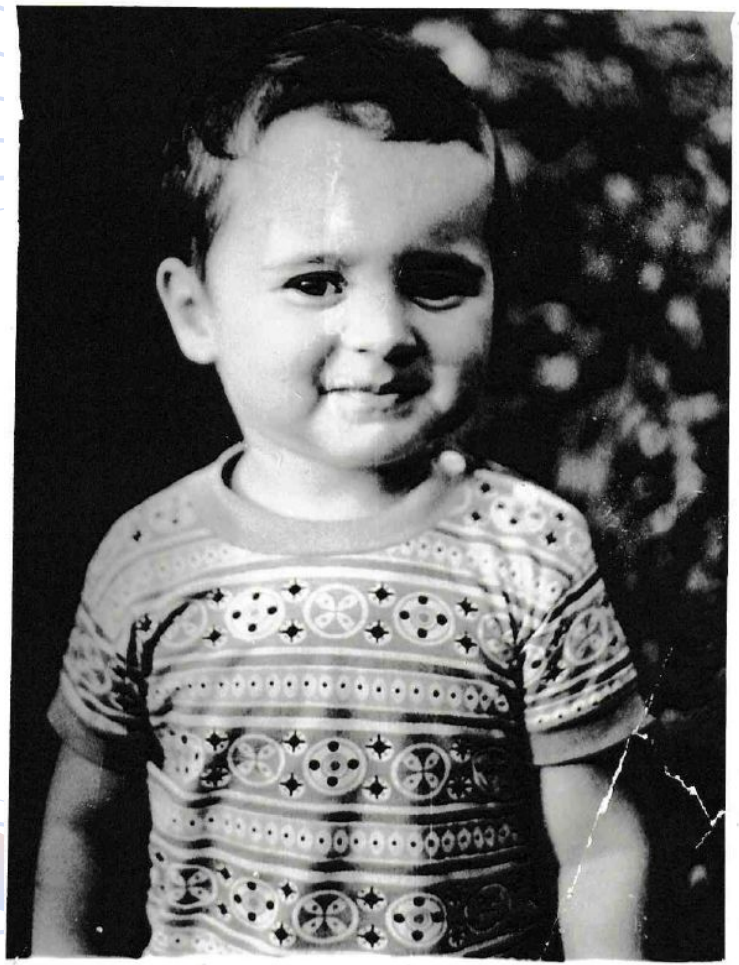
Vladimir Yu. Protasov

Sirius, 13.07.2021

by Tatyana Zaitseva

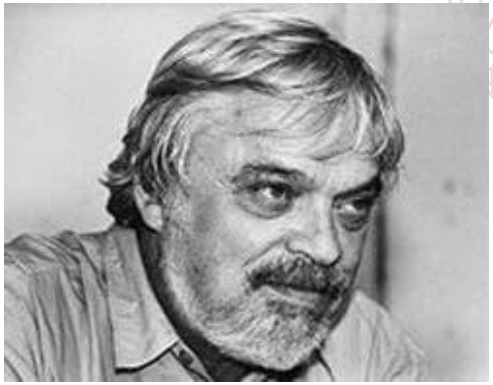


1973

1986

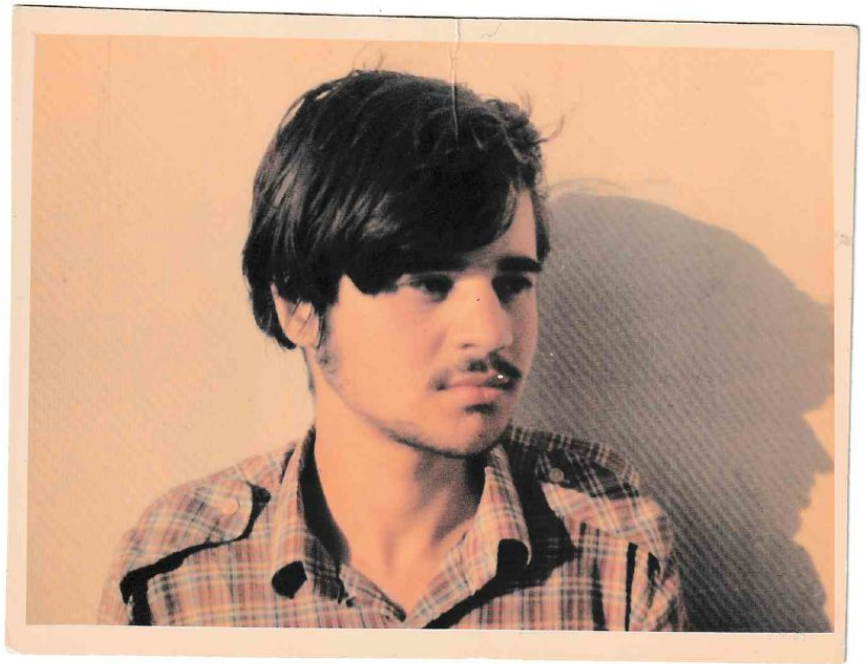
From childhood to school

S.V. Konyagin



I.F. Sharygin

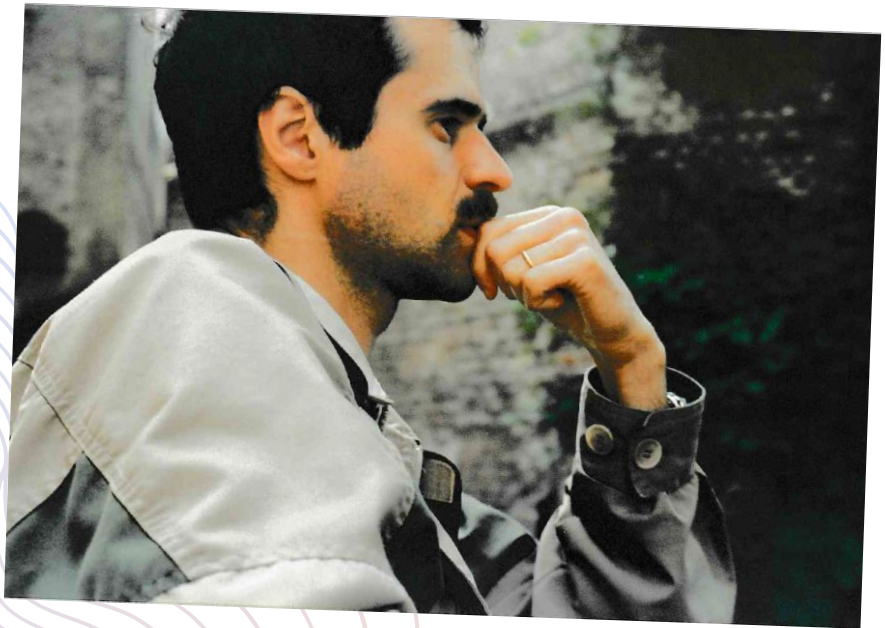
V.M. Tikhomirov

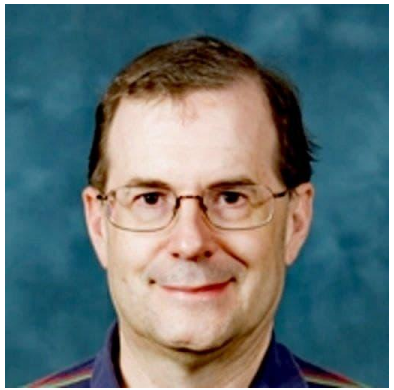


Princeton

Ingrid Daubechies

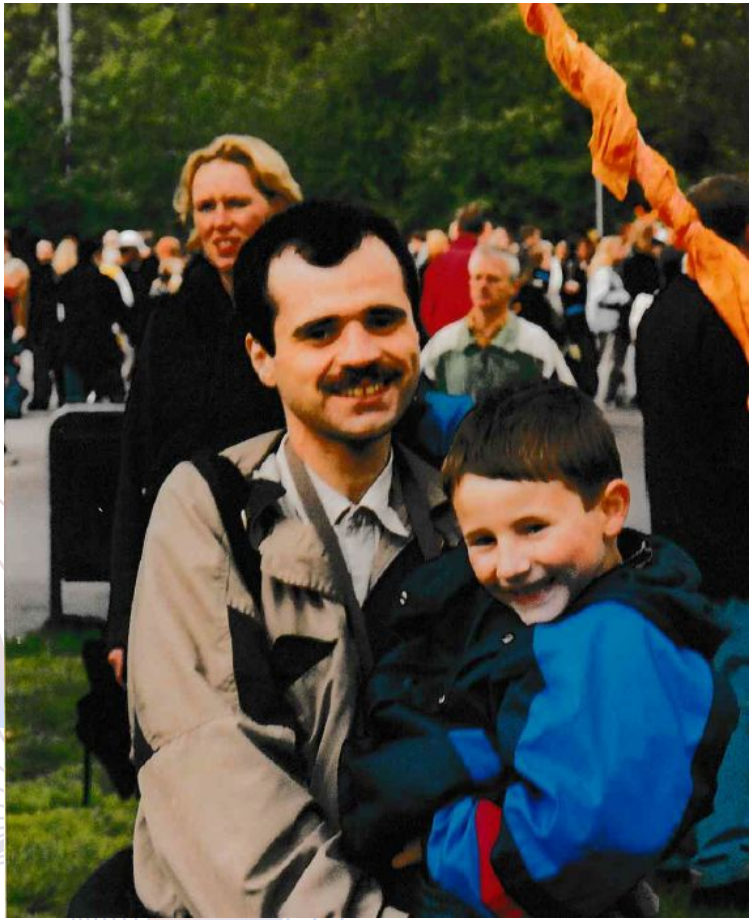
Jean Bourgain



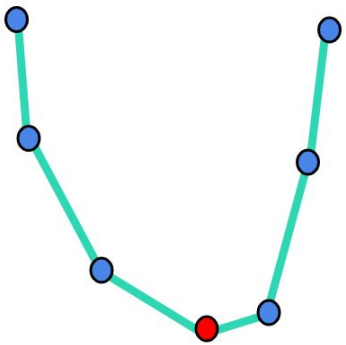


Jeffrey Lagarias

Amsterdam, 2001



with MSU colleagues



Derivative-free convex optimization

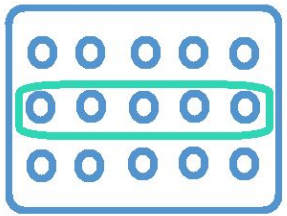
Joint spectral characteristics of linear operators

- Joint spectral radius
- Lyapunov exponent
- p-radius

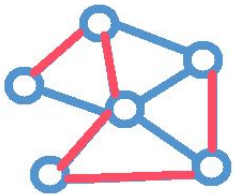
Optimization and numerical analysis

Optimizing the spectral radius

$$\lambda_{\max}(A) \to \min / \max$$
$$A \in \mathcal{M}$$



The problem of max acyclic subgraph



PDE (liquid flows)

Exact Computation of Joint Spectral Characteristics of Linear Operators

with Nicola Guglielmi, 2013

$$\mathcal{M} = \{A_1, \ldots, A_m\}, \quad \mathcal{M}^k = \left\{A_{d_k} \ldots A_{d_1} \mid d_j \in \{1, \ldots, m\}\right\}$$

$$\hat{\rho}(\mathcal{M}) = \lim_{k \to \infty} \max_{B \in \mathcal{M}^k} \|B\|^{1/k}$$

The algorithm for the JSR finds the exact value for the vast majority of matrix families in dimensions ≤20.

It was applied to give answers to several conjectures in combinatorics, number theory, and formal language theory.

The generalized joint spectral radius. A geometric approach

$$\mathcal{M} = \{A_1, \ldots, A_m\}, \quad \mathcal{M}^k = \left\{A_{d_k} \ldots A_{d_1} \mid d_j \in \{1, \ldots, m\}\right\}$$

1997

$$\hat{\rho}(\mathcal{M}) = \lim_{k\to\infty} \max_{B\in\mathcal{M}^k} \|B\|^{1/k}$$

$$\widehat{\rho}_p(\mathcal{M}) = \lim_{k\to\infty} \left(\frac{1}{m^k} \sum_{B\in\mathcal{M}^k} \|B\|^p \right)^{\frac{1}{pk}}, \quad p < \infty$$

The Dranishnikov-Konyagin theorem on existence of invariant convex bodies is extended in terms of the operation of generalized addition of convex sets.

The problem of calculating p-radius for even integers p is reduced to determining the usual spectral radius.

Computing Closest Stable Nonnegative Matrix

with Yu. Nesterov, 2020

The problem is to find the closest stable matrix for a positive dynamical system with discrete time.

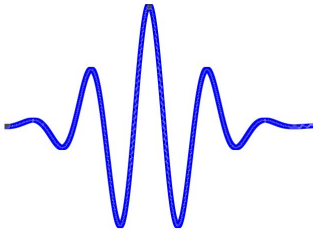
Max-norm – an exact solution of the corresponding nonconvex projection problems in polynomial time. ℓ_1, ℓ_∞ – also efficiently.

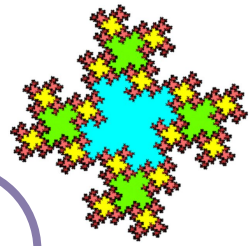
For all these three norms – exact descriptions of the region of stability around a given stable matrix.

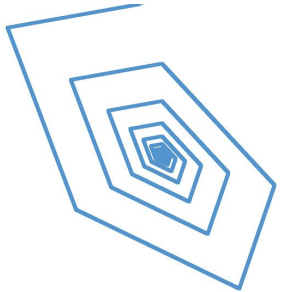
A new method for approximating the maximal eigenvalue of a nonnegative matrix: the local quadratic rate of convergence + polynomial-time global performance guarantees.

Functional analysis

- Functional equations with contractions of the argument
  - Hölder regularity
  - fractals
- Wavelets
  - dyadic wavelets
  - multivariate Haar
- Subdivision schemes
  - de Rham curves
- Random power series
  - Bernoulli convolutions
- Dynamical systems
  - on graphs
  - linear switched systems
  - stability
  - population models
- Reproducing kernels







Refinement Equations with Nonnegative Coefficients 2000

$$\Phi(x) = \sum_{k \in \mathbb{Z}} p_k \Phi(nx - k) \qquad p_k \geq 0 \qquad \sum_k p_k = 1$$

The equation has a unique* solution with bounded variation that is either absolutely continuous or purely singular. The criteria for separation of these two cases is proposed.

$$\varphi(x) = \sum_{k \in \mathbb{Z}} c_k \varphi(nx - k)$$

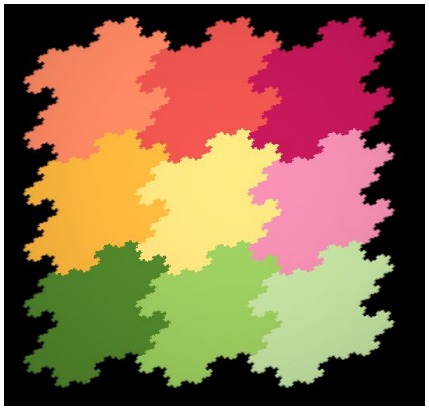
$$c_k \geq 0$$

$$\sum_k c_k = n \qquad \sum_k |k| c_k < \infty$$

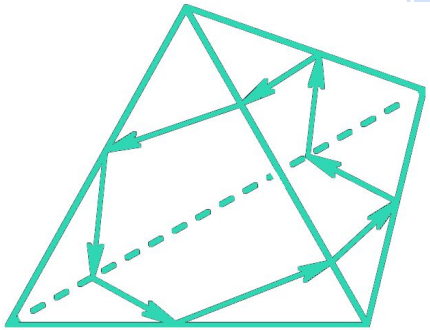
A criteria for existence of L1-solutions

Geometry

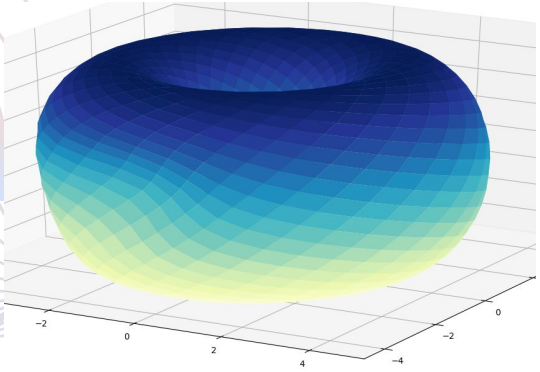
Tilings of spaces

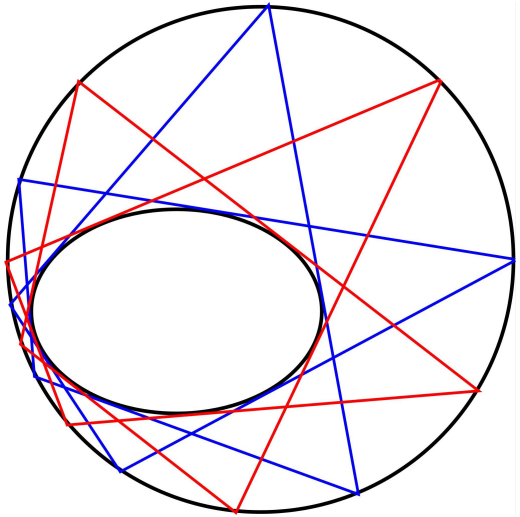


Geodesics on convex surfaces and on polyhedra



Design of curves and surfaces



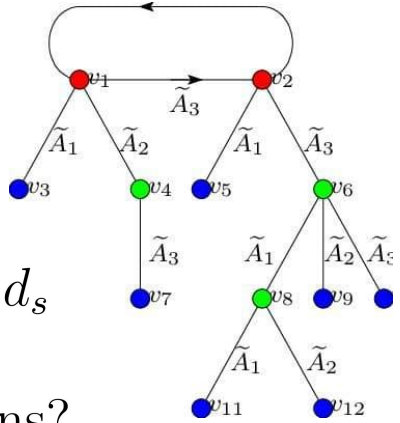


Poncelet-type theorems, invariant measures of conics

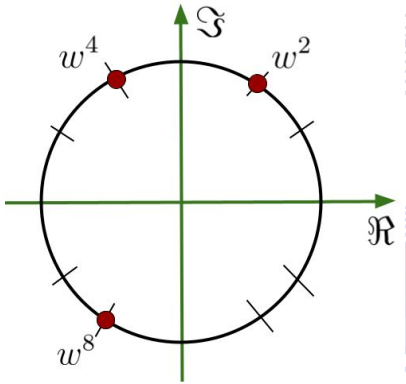
Euler binary partition function

$$k = d_0 + 2^1 d_1 + \ldots + 2^s d_s$$
$$d_i \in \{0, \ldots, n-1\}$$

The number of expansions?



Cyclotomic polynomials



Combinatorics and number theory

Combinatorics of non-negative matrices

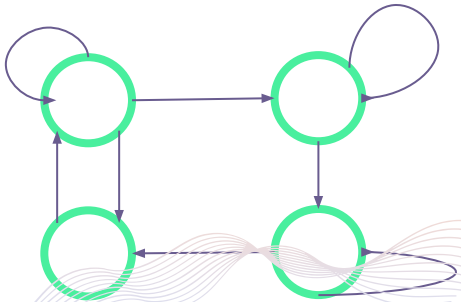
Theory of formal languages

01120120002...

Non-deterministic automata

synchronization

Černý conjecture



Sets of nonnegative matrices without positive products

The matrix A is primitive if for some N holds $A^N > 0$.

with A.S. Voynov, 2013

Primitivity conditions for one matrix are well-known. Primitive matrices are irreducible.

(Perron-Frobenius theory)

$$\begin{pmatrix} 0 & 0 & 0 & \dots & 0 & B_r \\ B_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & B_2 & 0 & \dots & 0 & 0 \\ \vdots & & \vdots & & & \vdots \\ 0 & 0 & 0 & \dots & B_{r-1} & 0 \end{pmatrix}$$

The imprimitivity index r of an irreducible matrix is equal to the total number of largest by modulo eigenvalues.

Generalization to a family of nonnegative matrices without strictly positive product of matrices of this family is obtained.

A polynomial algorithm to check this property and find imprimitivity index is suggested.

Some coauthors

Vincent Blondel

jsr, combinatorics

Nicola Guglielmi

numerical methods, dynamical systems

Maria Skopina

wavelets

Raphaël Jungers

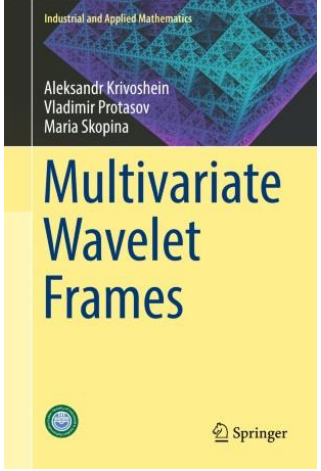
Lyapunov exponent, automata theory, nonnegative matrices

Yurii Nesterov

convex optimization, linear algebra

with
M. Skopina,
I. Novikov

Wavelet theory



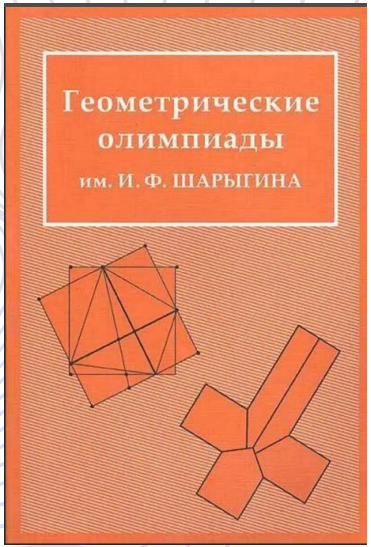
with A. Krivoshein,
M. Skopina

Multivariate
Wavelet Frames

Books-1,2,3

with E. Galeev.,
M. Zelikin, etc.

Optimal control



With A. A. Zaslavskiy,
D. I. Sharygin

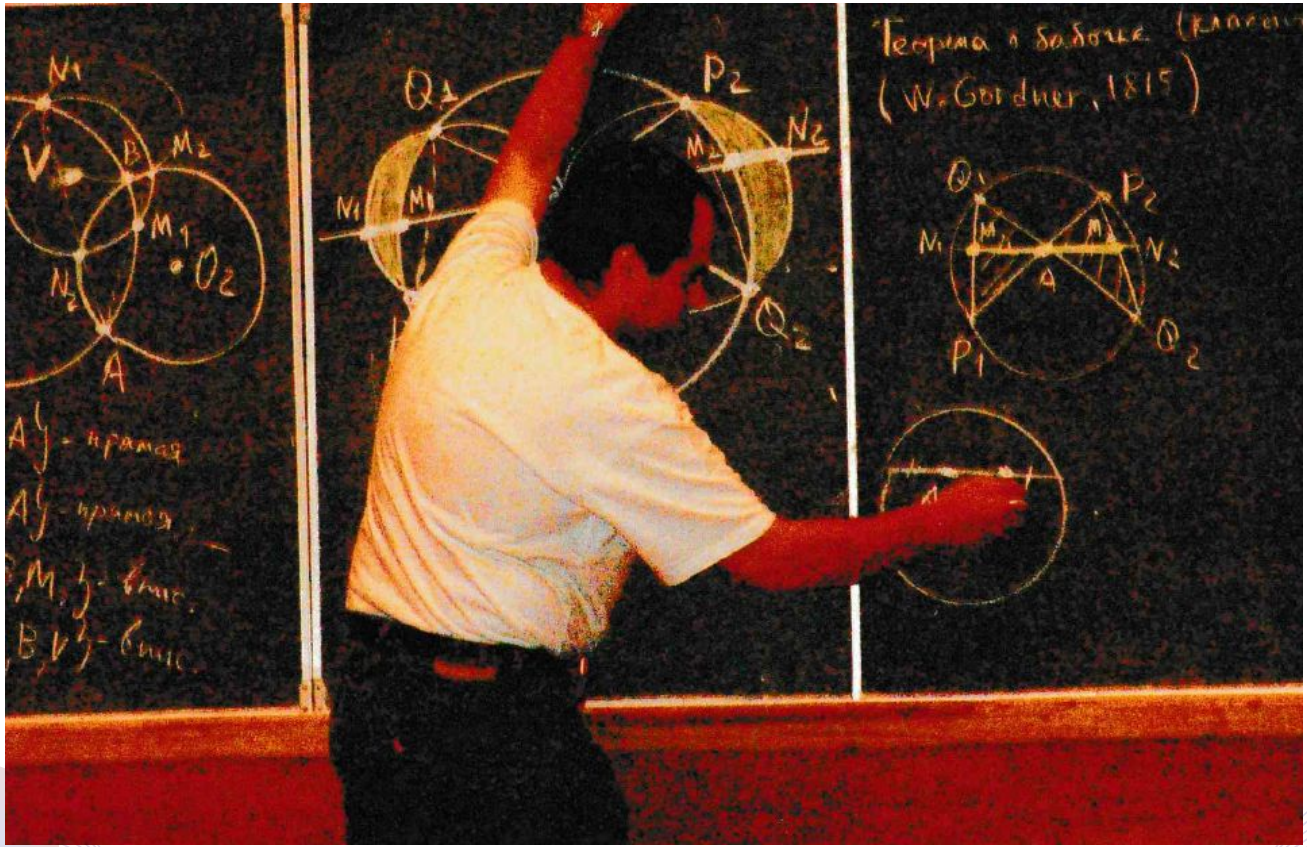
Sharygin's geometric
olympiads

Sinusoid
and fractal

Books-4,5,6

Maxima
and minima
in geometry

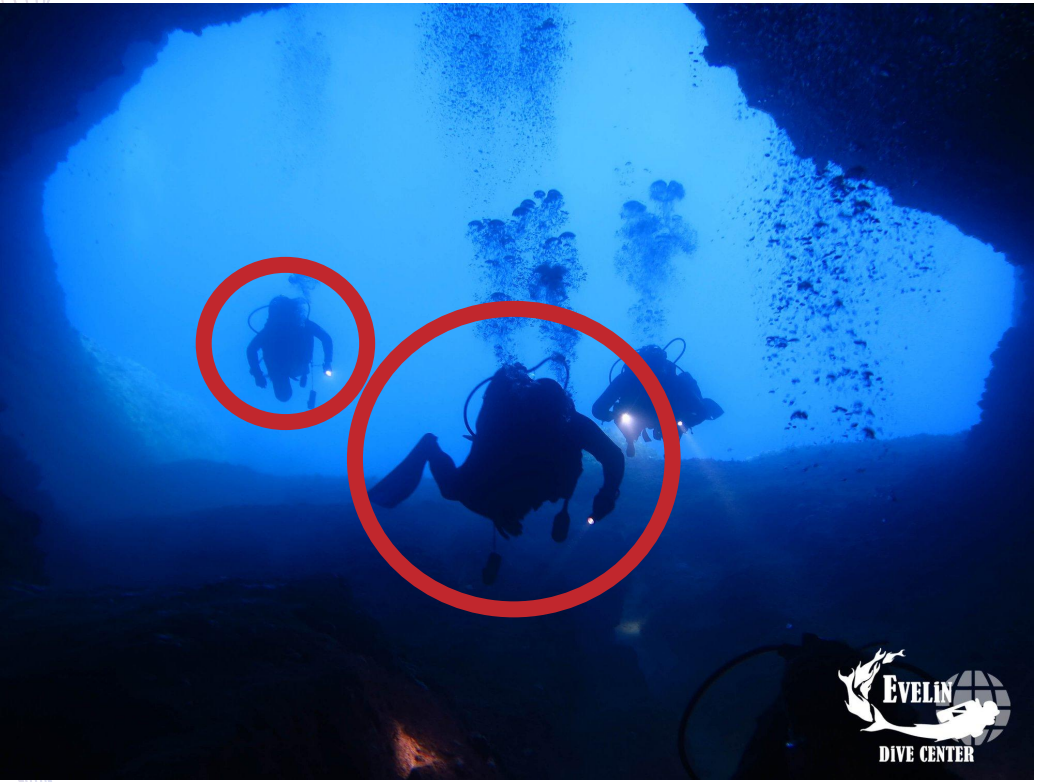
Dubna, Summer school “Contemporary Mathematics”



Children

Diving

Diving

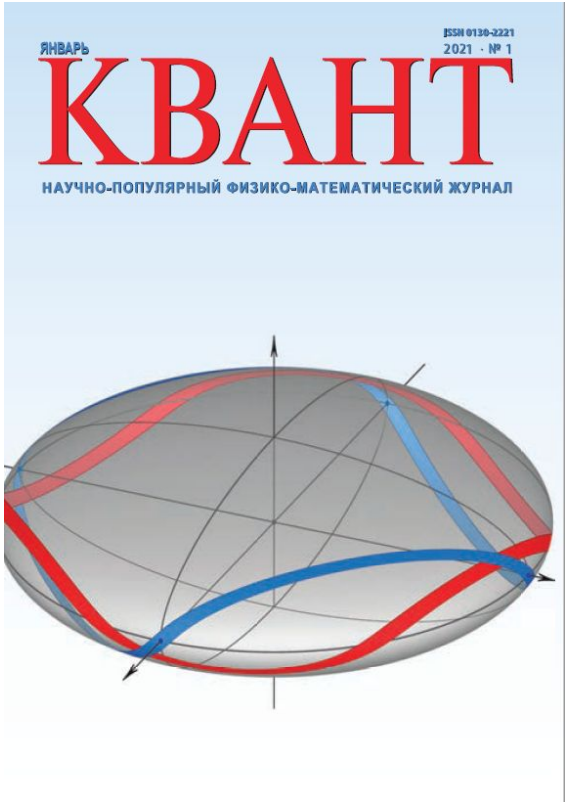


Hiking

Hiking

Photography

Papers in Kvant-1



Shortest paths and the Poincaré conjecture

no. 11-12 (2020), 8–12, link
+ no. 1 (2021), 12-22, link

How long does it take to dock?

With S.V. Dvoryaninov, Z. Krauter, no. 11 (2017), 2-9, link

The geometry of the stellar sky

no. 2 (2010), 14-22, link

How to get out of the forest?

With A.A. Zaslavskiy, no. 9 (2020), 10-17, link

Out into space-2

no. 12 (2017), 7-11, link +
no. 1-2 (2018), link1, link2

The Lp space and the remarkable points of the triangle

With V.M. Tikhomirov, no. 2 (2012), 2-11, link

An example from Kvant's paper "How long does it take to dock"

Our ship is coming. However, our docking takes an infinite time!

An example from Kvant's paper "How long does it take to dock"

1) How long does it take to dock if the velocity is f(x), where x is the distance between boat and shore?

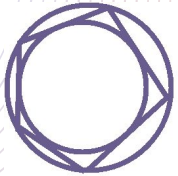
This is an optimal control problem!

If f(x) is differentiable at x = 0, it takes an infinite time.

2) A bug sits at the base of a bamboo of length 1 m. The bamboo is constantly growing by 1m per day. The bug crawls up by 1 mm per day (relative to bamboo). Will he ever reach the top?

Yes! However, the approximate time is 5×10^{431} years.

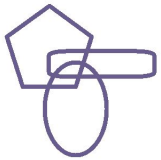
Papers in Kvant-2



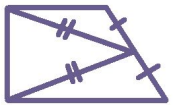
Two centuries of Poncelet's theorem

no. 5-6 (2014), 2-12, link

Helly's theorem and around



no. 3 (2009), 8-14, link



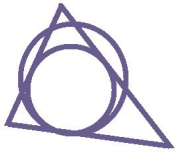
About two cyclists and a cherrystone

no. 3 (2008), 41-44, link

Tangent circles: from Tebo to Feuerbach

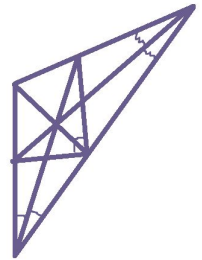
no. 4 (2008), 10-16, link

Around Feuerbach's theorem



no. 9 (1992), 51-58, link

Sharygin's geometric masterpieces



With V.M. Tikhomirov, no. 1 (2006), 35-39, link

Visiting Professor and Research positions

Institute for Advanced Studies (Princeton, USA)

Erasmus University Rotterdam,
University of Eindhoven (The Netherlands)

Paris-6 (Université Pierre et Marie Curie), France

CORE (Center of Operation Research and Economics),
UCL (Universite Catholique de Louvain) (Belgium)

University of Vienna, The Ervin Schrodinger Institute (Austria)

Sabanci University
(Istanbul, Turkey)

University of L'Aquila, University of Triest, GSSI (Italy)

Shanghai Jiao Tong
University, (Shanghai, China)

Hong Kong University of Science and Technology (Hong Kong)

Technion (Haifa, Israel)

Thanks!

Do you have any questions?

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v-protassov@yandex.ru

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